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IS MATHEMATICS TEACHING DEVELOPING LEARNER'S KEY COMPETENCES?

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Abstract. The goal of this paper is to try to answer following questions: To what extent does mathematics teaching develop key competences of students? Are trends of student's key competences development implemented in current education so that our school graduates stand the European competition? We searched for the answers in a survey and in this work we want to present the results of the study. In the introduction theoretical resources are described, the term 'key competences' is defined and system of mathematical key competences is outlined. In the next part of this paper we follow the survey offering answers to asked questions. Goal of the survey is presented, hypotheses are set and applied methods are characterised. We deal with survey realisation and quantitative and qualitative evaluation and we introduce possible improvements.

ZDM Subject Classification: D63; *AMS Subject Classification*: 00A35.

Key words and phrases: Key competences, categories of mathematical key competences, pedagogical survey.

Introduction

The more and more important goal of school systems is preparing scholars to successfully stand the information oriented society requirements as well as to take as many opportunities this society offers as possible. This goal leads to the inspection of the content of education, educational methods and goals which perforce instigates the interest in key competences. Education system aims more and more at knowledge, capabilities and skills application, at building positive attitude to a subject and learning. As a result most of the countries re-defined their educational goals paying attention to key competences and this appears to be the right and effective step.

1. Key competence term definition

Key competences present relatively new phenomenon in current modern pedagogy and education politics. To certain extent it determines the shape of basic pedagogical documents in the whole world.

Terminology used for labelling the phenomenon of key competences started to form in English speaking countries and got developed from the term *basic skills* through *competences* to the final *key competences*. Besides the term key competences other expressions are used: *new basic skills*, *life competences*, *action competences*, *methodological* or *metacompetences*, etc. From the frequency perspective mostly used term is *key competences*.

What does the term *key competence* mean indeed? The term key competence cannot be considered to be strictly pedagogical or psychological expression. This phrase is used in technical as well as in common language from the beginning of 70s of 20th century. It got into the area of education in the late 90s of 20th century. Despite this no satisfactory and standard definition exists by now. Taking all characteristics and facts resulting from all area definitions from various specialists into consideration (see [1], [2], [4], [5], [9], [10], [13]) we get the following: *competences present the unification of all knowledge, skills, capabilities and attitudes. Individual competences enable their bearer act adequately in certain situation in specific field of activity.*

Key competences are those that are usable *not only in one but in more various fields of activity.* They present the part of knowledge, skills, capabilities and attitudes individual takes on during the whole life. (See Fig. 1).

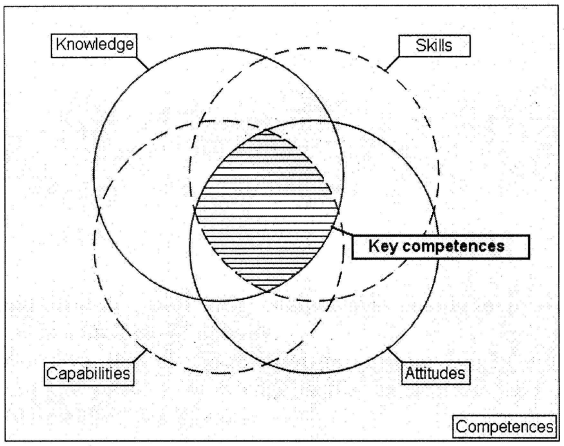

Fig. 1

Key competences could be held as multifunctional collection knowledge, skills, capabilities and attitudes and could be characterized as follows:

- *They have active, procedural character*: they are formed on the base of personal practical experience and activity and they are applied in praxis.
- *They form a complex unit*: they present unity of knowledge, skills, capabilities, attitudes and other elements that were perceived more independently by now.
- *They are dynamic and developed on various levels*: they change their quality during whole life. They do not date as information or expertise but keep their ability to develop (therefore they could be the base of lifelong education and personal flexibility).
- *They are the result of formal, non-formal and informal learning*: they are the result of lifelong learning. They are not the matter of personal performance only but they require favourable social and ecological surroundings.

2. Implementation of key competences into mathematical teaching

Identification and implementation of key competences into mathematical learning appear to be as big problem as is the key competences term delimitation.

Many methodological and practical problems go along key competences selection process. There is no worldwide accepted model in the field of key competences identification. In general three main streams exist. One of them takes the competences subjectively, i.e. it covers system of competences that relate to specific discipline or school subject. The second one takes them super-subjectively (cross-curriculum) and the third one is combined. When identifying key competences super-subjective orientation took precedence. The development of key competences is not connected with certain content or subject but it relates to procedural aspect of curriculum. Further problem is that there is no well known literature that would deal specifically with the problem of key competences in mathematics teaching. In general key competences problems in Slovakia were compiled, among others, by I. Turek and R. Hrmo who used the analyze of nowadays and mostly perspective needs of our society to outline six areas of key competences [5], [15]:

1. informative,
2. perceptual,
3. cognitive,
4. interpersonal,
5. communicative,
6. personal.

Given areas are not hierarchically sorted. They are considered to be of the same importance. We developed these areas (according to [6], [8], [14]) in range of mathematics teaching into twelve categories. Specific mathematical categories are included among them because we combined super-subjective and subjective perception of key competences when creating this system. There is a strict line between given categories as there are mathematical key competences there that could be included in several categories. This competence model is dynamic and open:

1. Mathematics thinking and pondering.
2. Mathematical terms, facts[1], claims and procedures.
3. Use of symbolic, formal and technical terms, relations and operations.
4. Figuring and describing of mathematical objects and situations, representation.
5. Asking a question, problem determination and its solution.
6. Mathematical modelling.
7. Mathematical argumentation, proof.
8. Use of tools.

[1]Facts—axioms and postulates of a given theory

9. Communication.
10. Information handling competences.
11. Competence related to attitudes and value system.
12. Personal and interpersonal competences.

We will spend more time on some of the categories in the following sections of the paper when evaluating the survey.

Now several other methodological and diagnostic questions come to mind. How to develop key competences? How to judge and evaluate the level of the development? Problems solving is considered to be the most suitable mean of developing and diagnostics of competences in mathematics. So we concentrated to creating of suitable problems.

Not all competences could be represented by problems, mainly in affective area. On the other hand some problems represent several competences. This is one of the reasons why it is suitable to count given competence categories system into wider classes [8], [14]:

1. *Reproduction level competences* – reproduction of learned material, making routine calculations and procedures and resolving routine problems,
2. *Connection level competences* - integration, connection and simple spreading of known material, modelling, connection of more methods that are known to students,
3. *Reflection level competences* – developed thinking, argumentation, abstraction, generalization and modelling used for new unknown contexts, original mathematical attitude, connection of several and more complicated methods, the look inside a problem infiltration into the core of the mathematics.

As you can see simple levels are based on the type of cognitive needs that are necessary to solve various mathematical problems. Mathematical competences mentioned above need not to be included in one class of competences only. Classes form conceptual continuity, beginning with simple reproduction off acts and calculation capabilities, followed by skills to take over various sources when solving problem and finishing with "mathematization" of real-life problems. In this case we take the hierarchy as follows: needs (e.g. in form of mathematical problems) require class 3 competences are mostly more difficult than those that require class 2 competences what in fact doesn't mean that class 2 competences are necessary for all class 3 competences.

3. Key competences level diagnostics tools

It is fully logical and completely right to try to recognize the level of key competences along their development and not only during the education process. To be able to judge the level of key competences of a pupil he/she has to prove by a performance that he/she has this certain competence and this performance must be measurable. According to M. Romainvillea [11] level evaluation of the competences in fact means forming of believes based on deduction. Person who performs the

evaluation has to judge according to student's performance, what is the possibility that the pupil assumes certain competences.

D. Curtis and R. Denton [3] identified four general attitudes when evaluating the level of control of key competences:

- Holistic review of level of these competences by teacher.
- Review according to batch of various papers and projects of a certain pupil.
- Evaluation in range of real-world reminding situations.
- Evaluation using standard tools appointed for measurement of key competence level.

Pedagogical diagnostics offers number of methods. According to attitudes mentioned above we took the following out of this set:

1. *working sheet method* (various kinds of didactic tests),
2. *observation method*,
3. *dialogue method*,
4. *questionnaire method*,
5. analyze of scholars' papers,
6. other methods, according to the goal of probe.

According to the probe difficulty of attitudes that will often be the subject of our search it is not suitable, in most cases, to focus just to one method. More mentioned diagnostics methods need to be used. Man needs to gain knowledge about existing possibilities to be able to decide for a certain method or methods that are adequate for given problem. This selection is positively influenced by precise formulation of diagnostics goals and adherence of basic methodological rules.

4. Survey

Various monitor and survey results show that despite the amount of mathematics skills Slovak Republic scholars hold according to curriculum they do not have the capability to use these skills in real-world situation (see the results of monitors: PISA 2003, TIMSS 2003). According these surveys we asked questions said in the introduction.

The goal of the survey is to determine whether and to what extent current mathematics teaching develops or suppresses scholar's key competences and state the importance to concentrate to key competences, specifically their gaining and improvement. According to set goal we worded two hypotheses of this survey.

H_1: Current mathematics teaching *does not suppress nor develops* key competences of pupil at adequate level. Scholars develop mostly their reproductive level competences, they develop their connection level competences less and do not develop their reflection level competences at all.

H_2: Education subjects consider gaining and developing of key competences to be *useful* in one's present as well as in one's future. To verify our hypotheses

we used working sheets that diagnose the level of some scholars' competences and anonymous form questionnaires.

We used working sheets to verify hypothesis H_1 and questionnaires served to verify hypothesis H_2 and as complementary method to working sheets one.

4.1. Working sheets

201 elementary school scholars and 74 high school pupils were involved to working sheet testing in 2006. We tried to learn scholars' capability level to activate their mathematical competences to solve problems not only to find out their mathematical knowledge level by this testing. This attitude to scholar's mathematics capability level diagnostics is in contrast with traditional school testing that is usually narrower.

Working sheets problems are in full correspondence with high school mathematics curriculum. They are concentrated to geometry mostly because this area involves logical and abstract and functional thinking and also imagination and intuition. According to demands we divided the problems into three level classes mentioned above—problems diagnosing reproductive level competences (1st level), problems diagnosing connection level competences (2nd level) and reflexion level competences (3rd level).

Content subject of the testing were *communication competences* (the ability to express oneself to the question of mathematics content and argumentation and proof and ability to take notes in a symbolic way and to understand those notes), *information handling competences* (working with texts and images and capability to criticize information (veracity) and to evaluate information (their worth) and qualitative character information involved in diagrams and ability to modify information) and *problem solving competences* (capability to think and conclude logically and functionally and knowledge of terms and capability to apply them and knowledge of algorithms and procedures and ability to use them and mathematization and demathematization and representation and description and ability to define the problem and find fitting strategy and apply it and capability to use tools and have the information about the range of their usage).

We used composite scoring to evaluate the successfulness of working sheets. Following percentage success scale to verify hypothesis H_1 was set: $(0\%, 36\%)$ – does not develop, $(36\%, 54\%)$ – slightly develops, $(54\%, 84\%)$ – develops, $(84\%, 100\%)$ – markedly develops. These intervals follow the rate of problems requiring competences at certain level.

We present a problem from working sheet Triangles Congruity as an example.

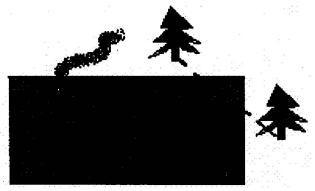

Fig. 2

PROBLEM 1. Determine the way to measure the distance of two trees if their direct join is not available (see Fig. 2). Use the figure and describe briefly the way to do this.

Solution and phenomena (competences) analysis. The most important key competence is the ability to use mathematics when establishing, formulating, solving and interpreting problems in various situations. The problem is appointed to the diagnostics of level of some problem solution diagnostics and some communication competences.

Solution. Our task is to find the way how to measure the distance of two trees if direct join is not available. We make a use of triangles congruity.

Sketch:

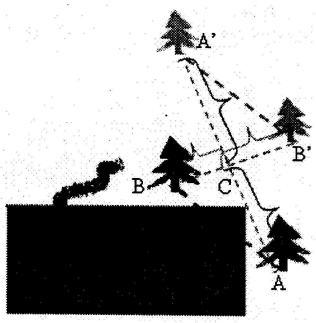

Fig. 3

Suggested solution description. We'll use e.g. two ropes of known and same length. We stretch the first rope from the tree A and the second one from the tree B the way they will cut themselves in halves. The ends of the ropes indicates the two points we called A' and B' in the figure. The distance between trees A and B is the same as the one between points A' and B'.

Proof.

$$\left.\begin{aligned} |AC| &= |A'C| \\ |BC| &= |B'C| \\ \angle ACB &= \angle A'CB' \end{aligned}\right\} \implies \triangle ABC \cong \triangle A'B'C \implies |AB| = |A'B'|$$

Tracked phenomena (competences).

- Determine basis (define the problem), find fitting strategy for problem solution – problem solution competences, reflection level. (2pts.)
- Figure usage, sketching – problem solution competences, reproduction level. (1pt.)
- Brief procedure description and solution accuracy – ability to express oneself in writing to the question of mathematics content, argumentation and proof – communication competences, reproduction and connection level. (3pts.)

Statistical evaluation results are presented in Table 1.

Competence	1st level	2nd level	3rd level
Communication	35.46%	17.93%	7.94%
Information handling	50.30%	44.72%	22.44%
Problem solving	58.98%	33.61%	10.45%
Total	52.26%	35.60%	11.94%

1st level – reproduction level competences; 2nd level – connection level competences; 3rd level – reflection level competences.

Table 1: Final evaluation of working sheets successfulness.

Received results analysis.

- Pupils do not have problems on the level of routine operations or problems where information is simply defined. They know how to use basic algorithms, procedures and how to find simple strategy for problems solutions. This indicates that scholars shouldn't have difficulties to solve problems according to instruction.
- Mathematics teaching does develop the problems solutions on the reproductive level. The other levels competences are not developed though.
- Scholars have little difficulties with problems where information is defined not that simply or where they need to use more sources of information. Our pupils lack some practical skills e.g. graph reading or figure reading and interpreting read information even if these are common information and explanation tools.
- Information handling competences seems to be slightly developed but not enough to reach adequate level.
- Scholars have bigger difficulties with problems where written description of the solution or result interpretation is required which relates to communication competences. Overall communication competences in competence rating came out as the worst (specifically *argumentation*). These competences show to be not developed.
- The outcome of these results is that the mathematics teaching does not develop pupil's key competences at adequate level. Even the reproductive level competences are average although in working sheets there were no problems they wouldn't meet during mathematics classes. Using this diagnostic method Hypothesis H_1 was proved.

4.2. Questionnaires.

Questionnaires were appointed to pupils and mathematics teachers at elementary and high schools. Respondents were given a chance to evaluate mathematics teaching from the point of view of developing of some key competences as well as a need of their gaining and developing by means of five value scale. A respondent assigned a value to each competence in a certain way using a set of values $\{-2, -1, 0, 1, 2\}$, where -2 means that mathematics teaching strongly suppress given competence, -1 means it suppress it, 0 – it doesn't suppress it, but doesn't develop it either, 1 – it develops it and 2 means that mathematics teaching strongly develops given competence. In case of evaluation of a need of assuming and developing of given competence respondent was given the same value scale where -2 means it is absolutely useless to assume and develop given competence, -1 means it is rather useless, 0 means I am not sure to decide, 1 – it is useful and 2 means it is very useful to assume and develop given competence.

Questionnaires were filled in April 2006 and survey counted 445 respondents, 89 teachers and 356 scholars. 46 teachers and 195 scholars were from elementary schools and 43 teachers and 161 students were from high schools (training institutions, technical schools and grammar schools).

43 competences in total were listed in questionnaires. Because of meaningly closeness of partial expressions we divided these competences into following six bigger groups: *communication competences, information handling competences, perceptual competences, problem solving competences, personal competences and interpersonal competences.*

Communication competences and information handling competences have several forms of demonstration. It is not only development of good phrasing capability but willingness to listen to other people and take their opinion into consideration. Quality and efficiency of subjects work with information sources is based on these demonstrations. Information handling competences exhibit mainly in ability to gain information and to ponder various data sources and criticize gained information.

All mentioned competences are closely bound to *perceptual competences, problem solving competences and personal competences* in range of which we prefer to organize self perceptual process and to be responsible for self learning, involvement, tenaciousness, ability to manage the insecurity and situation comprehensiveness, ability to solve problems and capability to find new solutions.

Interpersonal competences are the most important competences in the society. We meet their demonstrations daily and occurrence of most of them is essential. That means behaviour expressions like solidarity, empathy, tolerance even altruism, cooperation ability and team work. There are only few occupations where you don't need to cooperate with someone.

We set up following value scale for certain options to verify hypothesis H_2: $(-2, -1.5)$ – strongly suppresses, it is absolutely useless; $(-1.5, -0.5)$ – suppresses, useless; $(-0.5, 0.5)$ – does not suppress, nor develops; cannot decide; $(0.5, 1.5)$ – develops, useful; $(1.5, 2)$ – strongly develops, very useful. Statistical evaluation results are listed in the histogram on Fig. 4.

Gained results analysis.

- As expected, mathematics teachers are bit more optimistic in development of key competences perspective than are the scholars. Likewise teachers address the importance to the need to assume and develop key competences more than pupils.
- Mathematics teaching slightly develops *communication competences* while it is necessary to assume and develop these competences. Yet teachers do consider these competences to be very important. On the other hand according to the pupils' mathematics teaching does not influence these competences. However the school develops their ability to express themselves in writing it does not develop partial competences as to take part in a discussion and express their own opinion, to defend their own opinion, to listen to other people and consider their opinion or to present the results of their own work.
- According to respondents mathematics teaching does not influence information handling competences even though they consider them to be important. Pupils positively evaluated the development of *information assumption competences* that they consider to be very important as do the teachers. However, develop-

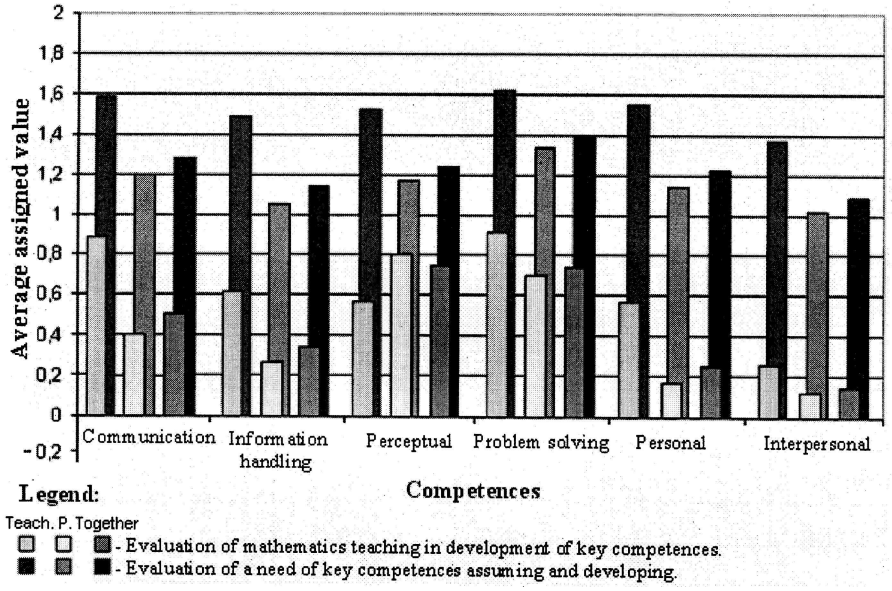

Fig. 4

ment of following competences was judged very negatively: to ponder various information sources (the most fitting information source selection), criticize assumed information and evaluate gained information. According to most of the scholars' mathematics teaching suppress mentioned competences even though they do consider them to be important.

- In case of *perceptual competences* (to organize self perceptual process and to be responsible for self learning) teachers' and scholars' evaluations do correspond. Yet the pupils evaluated the mathematics teaching more positively. According to respondents mathematics teaching develops these competences and their assumption and development is necessary.
- We count the following among the *problem solving competences*: ability to define the problem, to ask the question, capability to outline and describe; ability to experiment, to examine; capability to solve problems; to find new solutions; to make projects; to manage mathematical and model tools; to understand graphs, diagrams and tables; to use various tools and instruments. Pupils criticized mathematics teaching in the point of view of these competences. There were only two cases the value was higher than 1 and it was in case of following competences: to manage mathematical and model tools and to understand graphs, diagrams and tables. Most of the pupils evaluated mathematics teaching not influencing development of problem solving competences and that they do consider these competences to be important and teachers even consider them to be very important. The result in general was that mathematics teaching slightly influences development of problem solving

competences and their assumption and development are important.

- We included competences: to make decisions and ability to make use of experience that are also used in problem solving but are not organic part of the solution itself to the group of *personal competences*. Other competences like: to organize one's self work, to be able to work individually, to be tenacious in case of difficulties, ability to manage insecurity and situation comprehensiveness, to be critical, flexible by changes or to judge the risks influence the general ability to solve problems and that is why it is important and sometimes necessary to have these abilities. According the respondents evaluation mathematics teaching does not influence to the development of personal competences even though they are important. This conclusion corresponds with pupils' one. Teachers differ only in opinion of a need of assuming and developing of these competences. According to them these are very useful.
- *Interpersonal competences* got the worst result. Mathematics teaching absolutely does not influence these competences even though they are important. Yet it slightly suppress leader abilities and competence to show solidarity. It is closely related to the competence of ability to assert oneself. According to respondents mathematics teaching does not influence this competence.

The results of the survey in fact confirmed stated hypotheses where we suggested that current mathematics teaching *does not suppress but does not develop* key competences of a pupil at adequate level either (56.43% out of all respondents' answers were in interval of $(-0.5, 0.5)$ and 25.12% of solutions of working sheets were in interval of $(0\%, 36\%)$), and that assuming and developing of key competences is useful at present as it is in the future (78.37% out of all responses were in interval of $(0.5, 1.5)$). Teachers even consider development of key competences to be very important, so do most of the pupils.

We even verified hypotheses by the Wilcoxon signed-rank test (the one-sample variant) (see [12], [16]) at the relevance level of $\alpha = 0.01$. Hypothesis H_1 was not rejected at the relevance level. That is why we consider it to be true. Hypothesis H_2 was refused at this relevance level in favour of alternative hypothesis that could be interpreted the following way: "Assuming and developing of key competences is more than useful in the one's life". That in fact does not challenge given hypothesis H_2.

Teachers' questionnaire shows that key competences problems are not really in pedagogical society attention centre, not even pedagogical science deals with this problems at adequate level. That is why forming of key competences at present is only an occasional phenomenon and it is just kind of minor product of our education system. Only one teacher out of 89 gave positive answer to the question no. 4: "Have you had a chance to learn about key competences problems?".

After survey realization during dialogues with respondents most of the commented was the naming of the competences that the names are too long and complicated and hard to understand for larger group of pupils. This was considered to be the imperfection of the questionnaires.

Conclusion

Development of key competences is a new phenomenon that could help to find the answers for the questions related to status and to the future of education. A need to equip young people with necessary key competences and to better the level of achieved education presents a necessary step when society prosperity escalation even according to Europe Parliament and Counsel of 18th December 2006 [7].

In the end we present two more reasons why it is important to deal with key competences problems in mathematics teaching:

1. According to our opinion it is possible to change the status of mathematics respecting today's and future needs of the society, so it means to re-define the curriculum of mathematics teaching only using the paradigm of key competences.
2. Our existing experience show that mathematics teaching respects key competences development and builds good base for whole pupil structure development.

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