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NUMBERS AS VISIBLE SHAPES

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Abstract. The aim of this note is to stipulate requirements for number pictures (corresponding manipulatives) in order to make them convenient for visual representation of numbers and arithmetic operations. These requirements are: uniform representing of the unit of counting, respecting of counting by one, projecting of the number value at the first glance, stability of the shape of arrangements, easy way of drawing or completing of arrangements by children, possibility to represent numbers and operations within larger blocks.

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Numbers are visible shapes, says R. Arnheim in his sophisticated book "Visual Thinking" (Faber and Faber Limited, London, 1970). This is really true for all numbers from the initial blocks. Namely, when we hear a number name or we read certain data; we become aware of some groups of ones, tens, hundreds, etc., although this reaction is completely spontaneous and we are barely conscious of it. On the other hand, very large numbers are not accompanied by such awareness and their meaning is fixed only by their long decimal notations.

In ancient civilizations numbers were represented (what also means denoted) in the form of specific arrangements of uniform signs. For example, in Babylon, by cuneiform signs, in Egypt, ones were represented by sticks, tens by arcs, hundreds by "curls", etc. Also, in didactics, several systems of number pictures are used to represent visually numbers up to ten. First example of such a system was suggested by Russian philanthropist Busse (Fedor Ivanovich Busse, 1794–1859), Fig. 1.

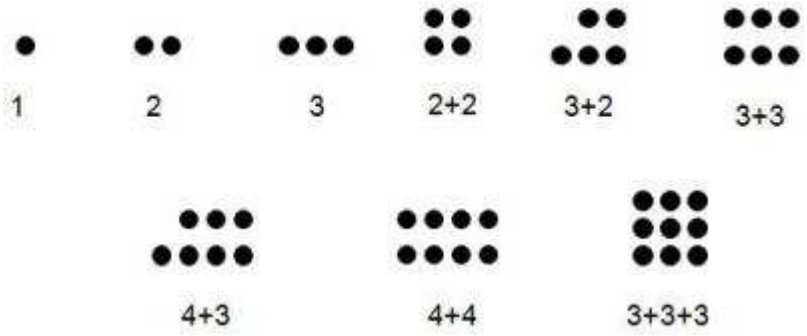


Fig. 1

Both, ancient arrangements and those in didactics had to be deliberately structured in order to project the corresponding number at the first glance. Contrary to it, when objects are chaotically grouped and when their number exceeds five, then, without counting, it is usually hard to tell their exact number. Even when the objects are regularly ordered to follow one after another, such an arrangement will be of little help. Such monotonous arrangements lack the right structuring and therefore the effect of such ordering is too small. But when we cast a look at Busse's arrangements we see that their structuring suggests sums (written below each of them) which are immediately recognized and calculated in almost no time.

A very suggestive representation of numbers is seen on domino pieces, where the spots are arranged into easily remembered shapes which immediately project the number values. Traditionally, teachers managed to get along using natural didactical material (grains of corn, beans, etc.). Children used to compose and decompose such arrangements of grains representing in that way numbers and operations with them (and they certainly had quite a lot of trouble with instability of such arrangements which might have been destroyed by a little uncontrolled move of the hand).

Here, we shall confine our considerations to number pictures as the way of visible representation of numbers and operations. Such a way of representing is also feasible in case of numbers belonging to larger number blocks and it can be carried out easily using a variety of printed material and by means of computer animation. In addition, teachers have to get rid of a widely spread prejudice that representing numbers by natural objects is better. Contrary to it,

- when less noise is present, learning effects are better.

And when the objective is development of calculation skills,

- once chosen way of representing numbers should not be changed and it should be applied throughout all didactical procedures.

Each way of visible representing of numbers and operations should meet the following requirements:

1. **The unit of counting is always represented by the same sign.**

This means that number pictures or, as we will often call them, arrangements of signs are patterns composed of uniform signs: bold dots, circlets, the straight line segments of the same length, etc.

2. **The way of arranging respects counting by one and emphasizes the role of numbers 5 and 10.**

To respect counting means that when an arrangement represents a certain number, by adding or removing one sign, the new arrangement will represent the following number or the preceding one, respectively. Turning attention to the Busse's arrangements we see that, for example, by adding a dot to the arrangement representing 3, the arrangement representing 4 is not obtained and the same holds in the case of numbers 8 and 9, etc.

As for the role of numbers 5 and 10, let us quote the well known linguist Jacob Grim (1785–1863), who says: "Counting begins by counting fingers". In this way

he points out the role of anthropomorphic apparatus that we have on our hands. On the other hand, in Slavic languages the change of declination suffix in nouns starting with 5 bears witness of a prehistorical number system having 5 for its base. A rod from Paleolithic period with cuts in groups of five up to 25, found in Veshtonice (Moravia) is a further evidence of that fact. Let us also remark that the arrangements of objects put in line, one after another meet the first of these requirements but not the second.

In Fig. 2, the number pictures that we will accept with preference represent numbers up to 10. This system of representing numbers will be called the arrangements of sticks. (The segments resemble sticks).

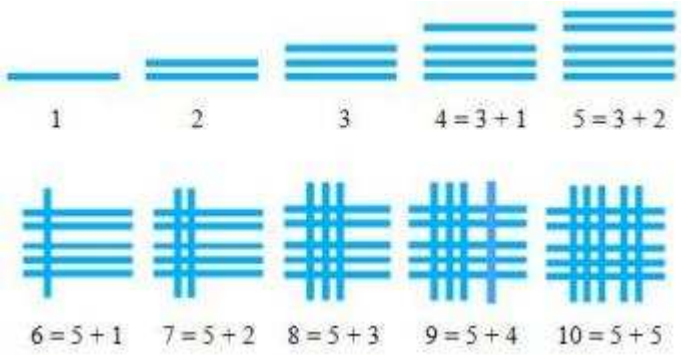


Fig. 2

3. Arrangements project number values at the first glance.

This means that arrangements are easily recognizable shapes having a distinct structure which is in accordance with decimal notation and the place of numbers 5 and 10. Their didactical role is to be visual notations for numbers alike those used in ancient civilizations.

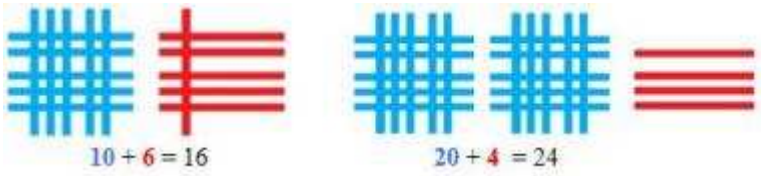


Fig. 3

4. The shape of arrangements is sufficiently stable.

When the signs representing the unit are drawn with some irregularities, (their size is not even and their places in an arrangement are disordered), then such a pattern may look chaotic loosing the function to be quickly recognizable. Some

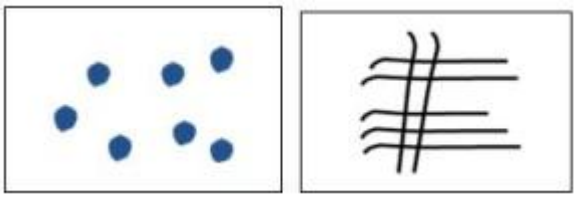


Fig. 4

systems are less sensitive (more stable) to such disturbances, while others are more sensitive (less stable).

5. Even children can draw arrangements easily (or complete them).

It is purposeless to force children to draw arrangements representing larger numbers. Such numbers should be represented by pictures already existing in the printed materials. But in the case of smaller numbers (up to 20) some drawing activities are desirable. In case of arrangements of sticks a matrix of dots should be attached (see Fig. 5).

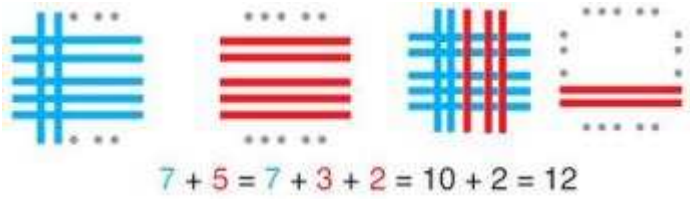


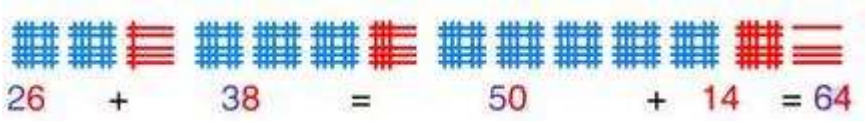
Fig. 5

Now the children are assigned to draw intervals by using a ruler and joining pairs of corresponding dots (by which the second part of Fig. 5 would result). In this way, forming from two arrangements the third one, the meaning of addition is also interpreted in pictures.

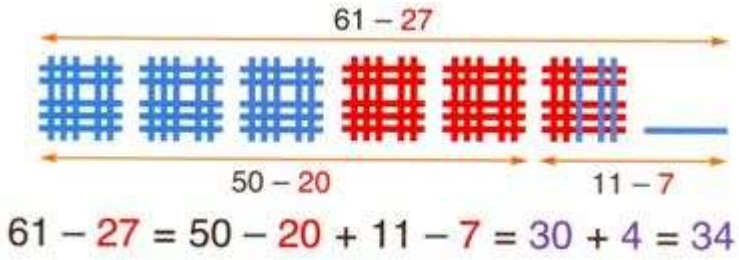
6. Arrangements can be used for representing numbers from larger blocks.

The pictures in Fig. 6 represent addition (under (a)), subtraction (under (b)) and multiplication (under (c)). Under (a), units of the two summands are in black, under (b), subtrahend is in black and under (c), the arrangement of sticks to be used in completing tens is in black.

(a)



(b)



(c)

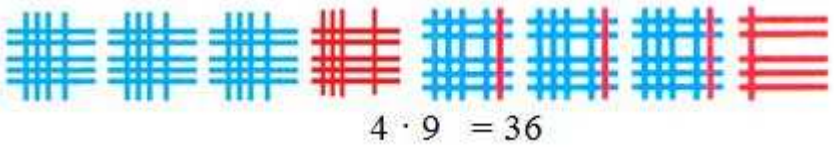


Fig. 6

At the end, let us say that the understanding of arrangements as a means of calculation is naïve. Their didactical purpose is different they exist to represent numbers as visible shapes and operations as the activity of rearrangement of such shapes.

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