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DANTE AND THE HUMANITIES

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Abstract. A course for interdisciplinary and integrated education dealing with Dante and his world is described and in particular the calculation of the birth and death dates of one of his ancestors using computer technology. Astronomy, mathematics, history, and literature (including the difficulties of translating Dante) were highlighted in teaching. The participants were students of the 119th High School in Sofia, Bulgaria. A brief review is made of the problems and questions relevant to this sort of education and especially those involved when teaching mathematics and the humanitites together. A detailed explanation is given of the problems, aims, and focus of the course. Also included is a description of the organization of the work, the content of the course, and the personal impressions of the authors. Finally, it is suggested that the dates normally presented by editors of Dante's *Divine Comedy* may be incorrect.

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Introduction

All of us accept the close relations between mathematics and the natural sciences. However, having made this assumption, perhaps in the field of education it is not as obvious as it might seem. A recent article on the subject complained that "[at] present there exists no clear definition of the meaning of integration of mathematics and science" (Lonning & DeFranco, 1995). This is by no means a new objection. Only a few years earlier the simple question had been posed "What does integration of science and mathematics mean?" The answer was that the nature of integration was in no way evident (Davison, Miller & Methany, 1995, pp. 226–230).

At the same time many people believe that the humanities are diametrically opposite to mathematics and the natural sciences. They suggest differences in the basic approaches and methods. It is often supposed that mathematicians, for example, are inadequate in the humanities and vice-versa. Many schools lend support to this supposition in that the pupils at a certain point must choose one camp or the other.

The school system of many countries underline this trend with special schools for mathematics and the natural sciences "opposed" by language schools and schools specializing in the arts. The universities and technical colleges, on the higher education level, no less reflect this situation. Education means specialization.

Nevertheless, one must ask, "Does education need [this] specialization?" The answer, of course, is "yes" but when does it become relevant? And where are

the limits? This forces a second question, "Is this divergence of the two aspects of our education system recommendable?" Do we want a future generation of mathematical and natural science "idiots" on the one hand and one of humanities "idiots" on the other? For educationalists the answer should be clear.

But, having accepted that the above is unacceptable, how can the school curriculum create the possibilities of making "attractive" links between the two disciplines by offering "interdisciplinary" and "integrative"[1] courses capable of showing the interactivity between them.

This was our point of departure. We wanted to see if we could develop a curriculum course satisfactory to/satisfying the needs of both sides. We decided that there would be many advantages if we started not in the 20th century but at the end of the Middle Ages. The reasons for this will become apparent. When we embarked on our Dante Project, we wanted to build a bridge between mathematics, astronomy, literature, history, religion and philosophy but not necessarily in that order. However, although the project was set in the Middle Ages, we wanted to use the modern technological possibilities to support it.

Why did we choose Dante?

1. Because Dante Alighieri (1265–1321) lived at the end of the so-called Middle Ages and undoubtedly was a product of them, he served as an example of what was later described as the "Renaissance man", one who excelled in all areas of knowledge (including the martial arts) and who employed them very effectively in his *Divine Comedy* which is regarded as representing the quintessence of the closing stages of the medieval world. In it he combined both science and culture, both in their broadest sense.
2. Because he was forced to be a rebel against his own society for many reasons and this makes him a figure with whom students can more easily identify.
3. Because he was European in a sense that other great writers like Shakespeare and Chaucer possibly were not and the main streams of European culture also flow through his work.
4. Because we were fortunate to find in Dante a good focus point from which we could spread out in different directions. This, in itself, was of cardinal importance in holding the attention of the students.

The Goals

1. To show the common ground between the natural sciences and the humanities during this period and how this appeared in the *Divine Comedy* where astronomy and literature, mathematics and history exist side by side in apparent harmony.

[1]Maybe it would be useful to notice what is usually meant by "interdisciplinary" and "integration". The former can be described approximately as "a knowledge view and curriculum approach that consciously applies methodology and language from more than one discipline to examine a central theme, issue, problem, topic or experience" (Jacobs, 1989), whereas integration deals with the nature of the relationships between the concepts from different disciplines.

2. To give the humanities students who normally view the *Divine Comedy* from the literature point of view a chance to see it from the natural sciences viewpoint and to show the natural scientists the literary value of the work.
3. To give mathematics and science students the opportunity of finding problems of common interest in Dante and to demonstrate that science and literature can be connected.
4. To show both teachers and students alike that an interdisciplinary approach can be fruitful and rewarding, thus integrating the various disciplines.
5. To show how modern technologies like computers, the Internet, etc. can be harnessed to assist teaching interdisciplinary material.
6. To give the students the atmosphere of another age or era, i.e. the Middle Ages where different rules and customs reigned, where a different set of daily problems existed from those with which we are confronted today depending naturally on where one was in the class hierarchy. Put another way, to give them a feeling for daily life in 13th century Florence.

The Focus

One of the most important things in such a project is knowing how and where to start, and how to keep up the students' attention during its course. It is necessary to establish a dialogue not just between teacher and student but between the students themselves. It was decided to introduce the students to a very specific problem in *Paradise*, the third book of the *Divine Comedy*.

Although there are many similar problems throughout the *Divine Comedy*, we decided to concentrate on a real scientific problem arising from the calculation of the birth date of Dante's great-great-grandfather, Cacciaguida which is featured in Canto XVI of *Paradise*, lines 34 to 39.

> His light said: From the day 'Ave' was said
> to that on which my mother, now a saint,
> heavy with child, gave birth to me, her son,
>
> to its own Lion this fiery star returned
> five hundred fifty times and thirty more
> to be rekindled underneath his paw. (Dante, 1986)

The "fiery star" mentioned above is Mars and, in clear text, these verses mean that from the day of Annunciation to Dante's great-great-grandfather birth Mars had passed 580 times through the constellation of Leo.

The date is not offered directly by Dante. Although parts of the *Divine Comedy* are autobiographical, no dates are offered anywhere. Cacciaguida's year of birth is given in the form of an "astronomical teaser". Past editors have generally agreed that the birth date is 1091 using Dante's "mathematics" (1.88 earth years is the duration of one Martian year and 1.88 multiplied by 580 gives 1091). However, this

date is not quite acceptable and is a subject of many scientific discussions. There are at least two basic objections.

First, because of the circumstances of the death of Dante's great-great-grandfather: it is known that he died fighting in the Second Crusade in 1147 at the age of 56. And second, it is known that the family name Alighieri has passed to Dante through at least one generation of women: Dante's mother; but since 1147 till 1265 (Dante's birth year) there are 118 years which gives at least 39 years for each generation—a number which is too large. On the other hand it was not unusual for old knights to go on the Crusades to purge their sins before death (according to the literature on the Crusades, many of them saw it as a last chance to atone for their sins and bring them a better bargain in afterlife). Life expectancy was much higher—depending of course on which strata of society one belonged—from the 11th to the 13th century than it was in the 14th and afterwards due to the devastating effects of the Black Death[2]. There was of course a high rate of child mortality, mentioned by Dante in the *Inferno*, and many died in the numerous inter-city and -state battles and skirmishes. However, many of Dante's contemporaries lived much longer lives than he did—his teacher Brunetto Latini died when he was 74, the painters Giotto and Cimabue lived to be 71 and 62 respectively, and Marco Polo was a septuagenarian when he died.

Why did Dante choose Cacciaguida?

Leaving aside the problems concerning Cacciaguida's birth date for a moment, if we ask *Why did we choose Dante?*, the second question is equally, if not more, so relevant. Otherwise we would have no "Cacciaguida problem". Seen purely statistically, Dante devotes 550 verses to his great-great-grandfather. Only his guide in *Paradise*, his beloved Beatrice and Vergil, his guide through the *Inferno* and *Purgatory*, earn more poetic attention. Dante has an innate respect for Cacciaguida, perhaps even views him as the founder of the Alighieri family, and he was dramatically important. Quite apart from the biography that Dante assigns to him in this contrived meeting, he represents for Dante a set of (moral) values that for him, Dante, do not seem to exist anymore. His great-grandfather is also mentioned although commentators claim that Dante gets his dates mixed up, and was still alive when Dante had already pronounced him dead. (He is "doing time" in *Purgatory* but there is no contact between them when Dante is passing through). The remaining members of his family get brief mentions or a cold silence. Why this is the case, need not concern us here. What does, is the birth date of Cacciaguida.

The Essence of the Problem

Dante used the Ptolemaic model in his astronomic calculations, for his descriptions of the planets and their motions, i.e. the geocentric model as opposed to the heliocentric one. However it would be wrong to assume that he had read

[2]A survey done on the population of Florence between 1327 and 1330 showed that 15% were over 60 years of age (Hevliky, 1997, p. 43).

Claudius Ptomely. Dante's reference work was *Elementa astronomica*, written by an Arabian astronomer, Alfraganus. The work was undoubtedly based on Ptolemy but it has been suggested that Dante never viewed the original opus. M. A. Orr goes as far to say that when Dante quotes Ptolemy in *Convivio*, he quotes him incorrectly (Orr, 1954, p. 149). Orr's *Dante and the Early Astronomers* must be regarded as the standard work in English on the subject[3]. For our purposes, it is interesting that the author reiterates the calculation given above for Cacciaguida's birth date. This has been accepted by generations of Dante editors and translators seemingly without question. However, she also mentions another interpretation of the text which had arisen from the attempts to get a later date for Cacciaguida's death) where "tre" or three replaces "trenta" or thirty. This would give the date as 1105 instead of 1091 (Orr, 1954, pp. 204–205).

She herself dismisses this possibility claiming it lacks scientific credibility. On the other hand, firstly given the always present likelihood of a transmission mistake by a scribe copying the manuscript (a common malady in days before the invention of movable type although even then there was plenty of room for error) and secondly because the new date gives perhaps a more realistic age to Cacciaguida when he went off to fight in the Second Crusade—he would have been 41 instead of 56—it seems a more reasonable proposition. Orr makes no consideration of either of these factors. As demonstrated below, our results offer other opportunities and pose new questions. More detailed explanation of the problem is given in (Author 2 & Author 3, 1997).

The above reasoning is based on theoretical calculation tacitly assuming the heliocentric model of the universe which was perhaps not known by Dante. Moreover these calculations do not correspond to the visible part of the sky and the trajectory of Mars on it and also do not take into account the following two facts:

1) During certain periods of time, the motion of Mars in the sky with respect to the stars is retrograde.
2) During one month every year the Sun is in the constellation of Leo and during this time Leo cannot be seen (moreover that Dante himself says "to be rekindled").

Calculating the time interval given by Dante more precisely is important because it gives us information about the notion that Dante's contemporaries have had about how many years had passed from Christmas and whether this notion differs from the date canonized with Gregorian Calendar and generally accepted today.

We shall briefly describe several models, which we use to investigate the data from above-mentioned Canto in *Paradise*.

The first model takes into account only 1), the second only 2) and the third

[3]There are, however, two standard works by Italian astronomers: Capasso I. (1967). *L'astronomia nella Divina Commedia*, Pisa. and Gizzi C. (1974) *L'astronomia nel Poema Sacra.* Two volumes, Naples. Also of interest is Richard Kay's (1994) *Dante's Christian Astrology*, Philadelphia, Univ. of Pennsylvania Pr.

both 1) and 2). In all these cases we interpret the lines 34–39 with the meaning that 580 is the number of times when Mars enters the constellation of Leo.

The fourth model takes into account 1) and 2), and 580 we interpret as the number of nights during which Mars is seen in the constellation of Leo. We consider a Cartesian coordinate system with a center in the Sun and x-axis directed to the constellation Leo (Figure 1). The Earth and Mars move around the Sun in circular orbits with radii respectively 149.596 and 227.94. We can assume that the Earth makes a full round for 365.24 days and Mars for 686.96 days.

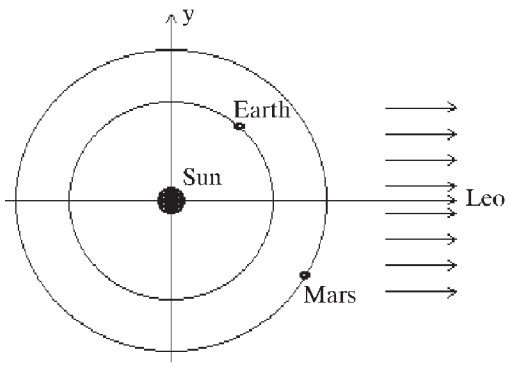

Fig. 1

For the first three models we accept that the constellation of Leo is a point at infinity. In this coordinate system Mars is seen from the Earth in the constellation Leo when the vector beginning at the Earth and ending at Mars is parallel and unidirectional to the x-axis (Figure 2). This last condition may be satisfied only if Mars is on the arc AB in Figure 2. Hence, with reference to the purposes of our investigation, at each orbit of Mars around the Sun, it is enough to restrict our attention only to its movement on the arc AB.

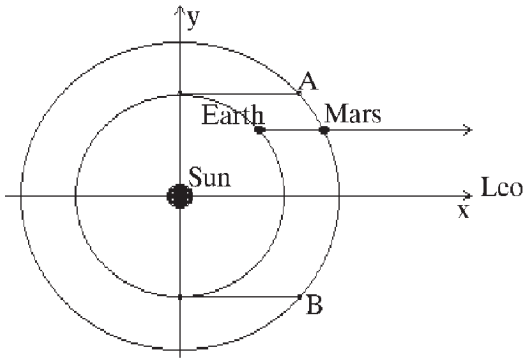

Fig. 2

Let us now consider a variable line $m(t)$ parallel to the x-axis through the point representing Mars. When Mars moves from A to B, the line $m(t)$ moves from

the position $m(a)$ to position $m(b)$ in Figure 3. When $m(t)$ coincides with $m(a)$ the Earth is "under" $m(t)$ and when $m(t)$ coincides with $m(b)$, the Earth is "on" $m(t)$. This corresponds to the fact that at this moment Mars is in the constellation of Leo. From these considerations we may conclude that, in every Martian year, equal to 1.88 earth years approximately, Mars is at least once in the constellation of Leo.

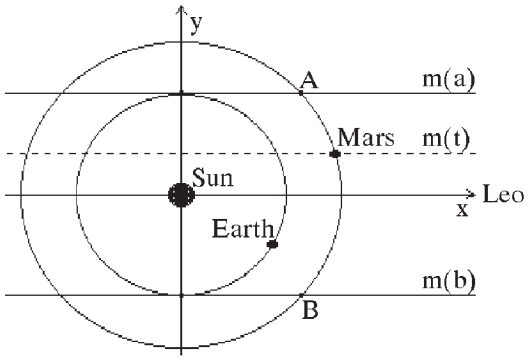

Fig. 3

If we accept that in every Martian year Mars is exactly once in Leo, then the information from Dante, that from Jesus Christ's birth to his great-great-grandfather's birth Mars has been in Leo 580 times, means that during this time 580 Martian years have passed. Therefore we would get the result that Dante's great-great-grandfather was born in 1091, the result stated by Orr. However, the assumption that in every Martian year Mars is exactly once in the constellation of Leo is not quite correct. Although it does not happen very often, in some years Mars is in Leo three times. For example, in Figure 4 are shown the respective positions of Mars and the Earth. It must be pointed out that this fact occurs only once because the linear velocity of the Earth in this coordinate system is larger than the linear velocity of Mars, i.e. it is conditioned by the specific relation of the angular velocities of Mars and the Earth in their movements around the Sun. However, for a sufficiently long period of time a considerable difference accumulates and its value, calculated by a computer, is about 80 years.

Organization of the Work

The course was held with a group of students from 119th High School in Sofia and was not a part of the curriculum. The school has a computer room with twelve networked IBM PC compatible computers which were almost completely satisfactory for our purposes. In this school, languages are emphasized and most of the students were from the English classes.

Everybody who was interested could attend the lectures and there were between 8–15 students depending on the lecturers; pupils, teachers, the Principal of the school and even the school janitor attended sometimes.

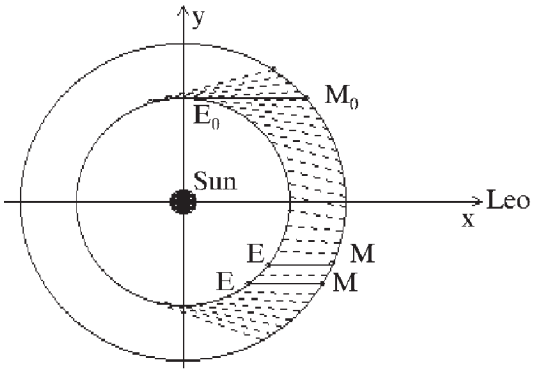

Fig. 4

The course program included 20 lectures, each of 45 minutes, on Saturday (which is not a school day in Bulgaria). Because of the demands of normal school sessions, Saturday was the only possibility for both the lecturers and the students. This was one of the weaker sides of the project but on the other hand those who came were devoted because they came in their own free time. Of course participation fluctuated but there was always a hard core of regular attendees. We tried, as much as possible, to give the students a mixed fare—first lectures and then practical exercises on the computers. The teaching staff numbered five: the three authors together with Evgenia Sendova and Ljudmila Baleva.

Schedule of the Course

The lectures given were as follows:

Introduction. In the brief preface to the series of lectures, the birth date of Cacciaguida was introduced as the problem around which the course was to be built. All the strings related to the solution of the problem were tied together only to be untied later in the separate lectures. The atmosphere of the late Middle Ages presented as Dante's and his contemporaries' daily life was considerably different to that of our own time: how time was calculated, the way the stars were observed, the importance of the Zodiac and horoscopes in people's lives, as well the general everyday life in Florence. Although, it must be added, that when Dante wrote the *Divine Comedy*, as well as his other major works, he was living in exile, never staying in any place for longer two years with the possible exception of Ravenna where he completed *Paradise* at the end of his life before he fell victim probably to malaria. *Paradise* included, of course, the Cantos dealing with Cacciaguida. The structure of the *Divine Comedy* and the problems in translating it were also briefly touched on.

Astronomy (4 lectures). Introduction to the basic astronomical notions and what was available to Dante when he was composing the *Divine Comedy*. This was to illustrate the astronomical data available in the "puzzle" under consideration,

for example, the structure of the Solar system, the development of the different ideas (geocentric and heliocentric), the motion of the planets in their orbits, an explanation of retrograde motion, the Zodiac constellations, Leo, and what does it mean Mars to pass through Leo, etc. Different maps of the star sky were shown with suitable explanations.

History and literature (8 lectures). a) Introduction to daily life in Florence during Dante's time—the population and life-expectancy statistics, the class structure, family life, the working day, the church, etc. Also considered were the different ways of measuring time—sundials, calendars, water clocks, and the church and monastery bells, and Dante's biography. An "interview" with Dante was simulated and students could ask "him" directly about the things they were interested in.

b) The structure of the *Divine Comedy*, its contents, the main characters. The problems translating Dante because of the rhyme structure and the different interpretations of Dante's verses, the mistakes connected with copying and translating old manuscripts of Dante as a best seller.

The students were given the task of searching Bulgarian literature and history for and of choosing their own unreal and real heroes to populate a Bulgarian *Divine Comedy*.

The lectures on history and literature were given in English which provided the students with additional language exercise. All others were delivered in Bulgarian.

Mathematics and computers (8 lectures). Actually there was not much mathematics in its "pure" form with the exception of some simple calculations connected with the duration of Martian year and multiplication by 580. But as far as we made an attempt to create a model of the Solar system on the computer screen, there was some more mathematics in "hidden" form: the planets became points, orbits became circles (we considered the circle as an approximation of the elliptical orbit which was good enough for our purposes), Sun or the Earth became centers of the circles (depending on the geocentric or heliocentric idea). Moreover, when trying to explain the retrograde motion (representing the "Earth view-point") we considered such curves like epicycloid and hypocycloid and demonstrated how they can be dynamically constructed on the screen.

The different computer models were made with the help of GEOMLAND—a computer plane geometry system environment created at the University of Sofia and used in many Bulgarian schools for teaching and learning plane geometry.

Small but (none the less) Important Details

What we wanted and what we got are, of course, two completely different things. We were unsure about what to expect. And naturally enough it turned out differently from what we had thought we would expect. But this is the purpose of running such pilot programs—to discover where the problems are and how they can best be solved:

1. The organization posed some questions which must be answered the next time the course is held.
2. Teaching a multidisciplinary course means keeping a delicate balance between, in our case, the humanities and the natural sciences. This is easier said than done and future attempts must be more carefully thought through.
3. No official attempt was made to get an active feed-back from the participants. Some came in unofficial conversations but unfortunately how the students felt has not been recorded for posterity.
4. There was post-mortem on the part of the lecturers immediately afterwards. Sometimes this can be positive but in our case it might have been better.
5. The choice of Saturday, as mentioned above, proved to be unfortunate and an eventual integration of the course in the normal course of studies should be considered.

Our Impressions of the course

Apart from the problems mentioned above which are all solvable, the question arises how, where and when such a course can be used? With a slightly improved organization and improved preparation, we think it is possible to teach this course at different levels, adapting the material suitable for the different age groups and the different education levels, whether it be university, high school or an even younger group of students.

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