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NEW IMPULSES FOR RESEARCH IN MATHEMATICS EDUCATION

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Abstract. We report about results of the scientific conference "Research in Mathematics Education" held in Belgrade under the organization of the Mathematical Society of Serbia, May 10–11, 2019. A survey of the presented investigations is followed by identification of common issues. Reports are classified according to the topics of research. Our discussion is accomplished with personal impressions about potentials for selected contributions to be implemented in schools and to serve as an impulse for further research in the field.

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1. Introduction

In an effort to follow the international trend of intensification of research in the field of mathematical education, the Mathematical Society of Serbia (MSS) took the initiative to organize a scientific conference devoted to this issue. The mentioned initiative should be considered as a continuation of previous endeavors of MSS. In the early '60's, an official document was accepted on a national level, containing an explicit recommendation for paying special care to mathematics education and development of the mathematical culture [14]. The MSS has pursued a wide range of activities, including publication of the scientific journal *The Teaching of Mathematics* and professional journal *Nastava matematike*, organized congresses of mathematicians, with a section on mathematics education; symposia and seminars (national and international) for professional development of teachers, etc.

In relation to math learning, math instructions, curriculum, assessment, teacher professional development and other related issues there are a number of critical moments and facts that require arbitration. This arbitration is quite often carried out with or without prior verification or incentive through research findings. Decisions regarding dilemmas are regularly made at the level of personal impressions of decision makers.

What does actually make curriculum developers recommend a particular approach for solving (in)equation or system of equations, over the other? One way is

to rely on the properties of operations and the other to use equivalent transformations. What are the reasons for evaluating one of these approaches as more "useful" or "suitable in particular grade" than the other one? We could ask on the basis of which findings happened a major didactical shift in formation of school geometry content matter from the "Hilbert-Euclidean" to "Klein transformation" approach. What "degree" of formal mathematics language should be used in any given grade? Research findings are quite often mentioned in relation to the levels of concept development (e.g. Van Hielle levels in learning geometric concepts). Yet, we wonder is it indeed too difficult or (in)appropriate for a 10-year-old pupil to learn about the concept of plane surface? At this age, pupils regularly study relations between straight lines. But, how to define parallel lines while avoiding the concept of plane surface? Should pupils learn mathematics by (re)discovery? The afore-mentioned are just a few examples of major decisions related to math instructions.

It is not an overstatement to say that research findings in the field of mathematics education either implicitly or explicitly have increasing influence on the way how mathematics is taught at the present time. There is a significant number of researchers who, among others, or exclusively, are engaged in research in mathematical education. It is evidenced by the number of numerous scientific meetings and articles in scientific and professional journals dealing with mathematics education. These are people of different profiles, levels of education and degree of affirmation at the national or international level. Should mathematicians participate in a more prominent way in community of researchers in mathematics education?

Given the orientation of the Mathematics Society of Serbia, the MSS applied for financial support from the European Researchers in Mathematics Education (ERME). This organization has a strong intention to promote the development of the field by expending the number of institutions, research organizations and researchers across Europe, especially in the region of Southeast Europe dealing with math education. Since the conference concept has been adopted, the Ministry of Education, Science and Technological Development of Serbia has also provided financial support for holding the event.

The conference "Research in Mathematics Education" took place in Belgrade, May 10–11, 2019.

Major aims for organizing the meeting were to introduce and exchange ideas and results of a scope of research in mathematics education. The themes of plenary presentations traced directions of the research and presented selected findings and perspectives on the proposed topic fields: 1) place and role of different participants in the research in mathematics education, 2) how to use lessons from history and research in educational practice, 3) research methods in mathematics education.

2. Plenary lectures

Jarmila Novotna in her opening talk "Bridging two worlds—cooperation between academics and teacher-researchers" acknowledged the wide scope of topics encompassed in the research in mathematics education involving teachers as

researchers. She presented the differences between the roles of teachers and researchers, the advantages of the links between both roles, and several models of the navigation of teachers of mathematics between theory and practice. Grounded in Brousseau's characterization she presented exemplars of successful teacher-research and collaboration among teachers and academics such as French COREM, CPS, and CLIL [6]. She recognized the advantages of teachers involved in research as they had the opportunity to exchange experience, to reflect on their own practice and improve the practice in their natural environment. She cautioned that answers to theoretical research questions usually do not have a direct impact on daily work of the teacher but that "action is linked with reflection" [17, p. 2].

Snežana Lawrence's address was focused on mathematics education as the founding principle of history. She proposed that "in order to construct a personal engagement and motivation for the learning of mathematics, learners need to develop skills for creating internal dialogues about mathematics" [10, p. 4]. The lecture was accomplished with a number of effective video clips from the educational practice.

Patrick Barmby in his plenary talk discussed different research methods he had used in research of mathematics education. Reflecting on his research interests such as visual representations and assessment, he compared three research methods: eye-tracking, comparative judgment, and Design-Based Research (DBR) methodology. He pointed to the need of finding a balance of both quantitative and qualitative approaches, as fitting to research questions and context. His workshop held within the Conference familiarized the participants with the comparative judgment methodology as they had the opportunity to be involved in the practice of comparative assessment of "children's understanding" via comparison of the overall impression of their productions.

3. Reports

Apart from plenary speakers, participants in the conference came from regional countries. Historically, these countries had similar mathematics curriculum over period of more than fifty years. Common for them is that they recently had or are currently in the process of reforming curriculum. Newly reformed curriculums inspired research of Barandovska and Ikodinović, Milinković, and Svetlik. Barandovska reported that Macedonian educational system is "under continuous revision and changes to the syllabuses and methods of teaching" throughout last decades. She analyzed the curriculum changes made according to the Cambridge International Examinations Center, giving a critical overview of the methods of teaching, topics covered by the curriculum and knowledge assessment and students results, before and after the changes. She presented the arguments for and against reforms, including quality of new textbooks as stated by teachers who are in the process of implementing the reforms [2].

Lipovec and Ferme examined the effects of training program of in-service primary grade teachers in Slovenia. On a sample of 104 teachers, they presented the evidence that a significantly larger number of participants shifted their focus

from general pedagogical aspects of knowledge to the mathematics-related aspects of their teaching performance [11].

Four out of fourteen selected papers focused on the assessment of math knowledge. Andrić & Mićić, Stanojević et al., Marić & Andrić, and Milenković and Dimitrijević presented research findings on the long term effects of learning. The first mentioned authors used the 100-year-old final exam in Mathematics for the year 1919. A sample of more than 500 high school graduation students, aged 18-19 participated in this study. The analysis of the achievements of the generations 2019 and 1919 showed that the achievements of 1919 generation had normal distribution unlike 2019 which had "anti-normal distribution" tendency since the peripheral features (max and min scores) were far more frequent than those that should be the most frequent in a normal distribution (medium grades) [13]. Mićić and Andrić remarked that hundred years ago mathematics literacy as expressed in the graduation exams was considered important. (Serbian distinguished poetess Desanka Maksimović proved to be excellent in mathematics.) In contrast, today the world at large, regardless of "technological age we live in" considers that it is not shameful to be mathematically ignorant or even math hater. This claim deserves attention of researchers as it could have important consequences for the math education of future generations.

Development of Standards of students' achievements in Mathematics on the national level proposed by a group of experts and practitioners have had a significant practical effect on the Serbian educational system in the last decade. Stanojević and colleagues reported on a large scale assessment of achievements of the final year of high school students in relation to the Math Standards. Their tasks were differentiated to reflect three levels (basic, intermediate and advanced) with aim to assess pupils math knowledge in accordance with the national standards. On a sample of 5806 students (approximately 10 percent of the population) it was found that pupils achievement had been below the expectations of the expert group who created the national standards. In most cases, pupils had poor performance in "application tasks" in which mathematical knowledge was to be applied. In after talk discussion a question was raised should the results of this research be used to make changes by lowering the standards in order to obtain "better results" or should the result alert policy makers that mathematics instruction should be improved.

Marić and Andrić concluded that moderate results obtained in their study based on a national "initial test" given at the beginning of high school should alarm middle school teachers about limited retention of knowledge of pupils by the beginning of new school cycle. It should also signal curriculum developers about which elements of mathematical curriculum were not well presented over period of eight years of elementary school to remain a part of long-lasted functional knowledge [13].

Contemporary conceptualization of mathematical competencies and mathematical thinking underline the importance of knowledge of multiple representations. Multiple representations and their role in learning mathematical concepts and solving problems was addressed in the studies of Ikodinović and his colleagues,

Dimitrijević and her colleagues, and Rakonjac and Milinković. The former examined the impact of the introduction of different representations of mathematical concepts in teaching on the development of conceptual understanding and problem-solving competencies. In the course of the research, a pedagogical experiment with parallel groups were conducted on a sample of 60 fourth-grade students, age 11, within the framework of the themes "Set N" and "Geometry". The effectiveness of the introduction of the experimental factor was evaluated by 1) a qualitative analysis of student responses (descriptors: procedural understanding, conceptual understanding), and 2) statistical analysis of quantitative data based on pupil responses in the final test. The criterion for assessing the understanding of a concept was not the knowledge of the term or verbal expression of its meaning but identifying the essential properties of the concept through problem-solving processes which involved comparing and/or establishing links between different representations of the concept. They concluded that the implementation of different representations of concepts in teaching effectively supported the development of conceptual understanding and success in problem-solving [19].

Newly reformed Serbian curriculum is based on outcomes. Ikodinović, Milinković and Svetlik focussed on the design of task using multiple representations as meant to assess the accomplishment of particular outcomes. Based on the theory of representations they designed sets of four matching representational contexts of tasks corresponding for particular outcomes (symbolic, verbal using math language, realistic, and pictorial) with a total of 24 tasks. The analysis of the student's work revealed extent in which representation of the problem used to evaluate student achievement of outcomes altered the results [9]. Pupils were more successful in solving tasks given in symbolic form or in verbal mathematical form than when the tasks was posed visually or in real context.

Mathematics teacher and mathematics textbook are primary sources of knowledge for pupils. Milenković and Dimitrijević examined the advantages and disadvantages of heuristic teaching in relation to traditional teaching results of two distinct teaching methods—teaching with the heuristic and elements of the problem-solving approach and traditional teaching of mathematics in elementary school. The experimental group of students was expected to come up with an appropriate rule for determining the area of the parallelogram through heuristic approach, with the help of manipulatives such as tangram and paper models. In the second group (the control group) the same teacher dealt with this mathematical contents in the usual, traditional way, teacher-centered classroom. Students were tested to examine possible differences in their understanding, theoretical and practical knowledge, depending on the teaching method. They found that pupils in experimental group had higher motivation, more active role, and extended training in problem solving. Yet, they found that "heuristic" method of instructions had some disadvantages: divergent students thinking is required, teacher had difficulty in creating a problem for each math class, "a hard problem can 'block' the class", and fewer number of tasks is solved during a lesson.

Lončarević reported on an experimental study designed to investigate effects

of teaching multiple strategies for problem solving [12]. He posed two questions: "Can using multiple strategies lead to greater gain in solving Linear Diophantine Equations (LDE), or does it lead to confusion?" and "What are the main reasons for using the specific method to solve LDE from students' point of view?" with 4 experimental conditions (Figure 1).

GROUP A	GROUP B	GROUP C	GROUP D
1. Algorithm for solving LDE with Farey sequences	1. Euclid's algorithm 2. Solving by guessing or inspection	1. Euclid's algorithm 2. Solving by guessing or inspection 3. Euler method	1. Euclid's algorithm 2. Solving by guessing or inspection 3. Euler method 4. Algorithm for solving LDE with Farey sequences

Figure 1. Four methods of teaching by Lončarević (2019)

The findings were that students who were exposed to multiple strategies for solving LDE became confused and developed resistance to multiple strategies. However exposure to multiple strategies helped each student to find "a method that works for him".

Mathematics textbook was a primary focus in the Nikolić, Orlić and Oparnica study. Their analysis was framed in the Skills-Properties-Uses-Representations (SPUR) multi-dimensional approach devised by Thompson and Kaur. The results showed that there exists a statistically significant difference in mathematical achievements between groups using different textbooks [16]. They remarked that there was no influence on the working experience of teachers and gender of students to mathematical achievement on SPUR test.

Dimitrijević, Popović, and Stanić examined the influence of the type of formulation of mathematical tasks on students' success in solving it. On a sample of 584 8th grade (15 years old) students they identified the problem of transition between different representations of the same problem remarking that all types of transitions are not equally included in school practice and that some transitions between representations are more difficult than others (e.g., verbal $\mapsto$ symbolic and graph $\mapsto$ symbolic belong to difficult transitions) [8].

Two studies at the conference were focused on the didactical tools used to support the learning of mathematics [7], [5]. Damjanović, Banković and Popović acknowledge the importance of the use of manipulatives in supporting and developing mathematical thinking. As they believe that a pre-condition for efficient and improved quality of mathematics teaching is the use of manipulatives they study teachers' competencies in using them. The results of the research identify parameters (e.g. perception of familiarity, capabilities) indicating teachers' competencies

and the probability that the manipulatives would be indeed successfully used in the classroom.

Two studies examined the opinions of participants in learning and teaching of mathematics. Barandovska discussed teachers' opinion about ongoing reform in North Macedonia. In the study, a sample of 857 university students majoring in different fields was questioned on their attitude toward mathematics [18].

Baranović also attended to undergraduate students. Her focus was on students math content knowledge. The study investigated the scope of primary education students' knowledge of quadrilaterals, the manner in which they establish relationships among quadrilaterals as well as students' misconceptions of these relationships. The results confirm earlier findings of difficulties of perspective primary grades teachers in identifying quadrilaterals in non-standard positions or of non-standard shapes, in addition to unexpectedly poor knowledge of quadrilaterals' properties and inability to establish inclusion relationships and hierarchy of concepts especially in the case of the trapezium [3].

Božić in a study focused on the learning of geometrical concepts: cylinder and three-dimensional view. He explored the possibilities of applying a three-dimensional view of geometric objects within the dynamic software *GeoGebra*, during the teaching and learning elements of stereometry (circular cylinder) to 15 years old pupils (8th grade). The students working in small groups had opportunity to independently create dynamic work-sheets, by using *GeoGebra* software, which they used for studying a three-dimensional view of solid figures (cylinder and prism), analyzing and comparing their elementary properties. Božić notified positive effects of using the software [5].

4. Concluding remarks

There is a necessity for mathematicians, professional educators in mathematics, to actively contribute within the field of research in mathematics education. One reason is that their subject matter knowledge helps identifying critical issues in mathematics instructions as well as benefits, drawbacks or alternatives in terms of educational outcomes we could achieve. Mathematicians can recognize importance of questions and whether it is methodically and mathematically justified. Resolution of the identified issues may bring advances with focused research agendas.

Universally, we live in the age of great democratization of the educational field, including mathematics education. Mathematics knowledge is assertively considered as of critical importance for today's children and as a consequence mathematics educations is under great public scrutiny. At the same time, mathematicians appear to have a less of a major role in it. They are surely minority in the community of researchers in the field. As a result, their position and expertise in the field of mathematics education is undermined. About the consequences of such developments, it is maybe too early to discuss. We believe that currently weak connection (sometime antagonism) between mathematicians on one side and professional educators, who are even without necessary mathematical background and education policy makers on the other side should be strengthen.

Here we identified major topics of interest for mathematicians which were stimulated to participate in research in mathematics education. Those were: design of tasks, problem-solving strategies, assessment, methods of teaching (with or without technology) and effects of curricular reforms. We anticipate that they would find interest to participate in forthcoming research in these and other topics.

We discussed the research findings presented at the conference talks as they illuminated aspects of teaching and learning of mathematics and their implications. We payed particular attention to points made as incentives for future research. This would additionally affirm results of the scientific meeting. Detailed program and the Book of Abstracts is available on the site of MSS: `https://dms.rs/`.

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