
OBITUARY

ACADEMICIAN MILOSAV M. MARJANOVIĆ

(August 24, 1931–May 9, 2023)



Brief academic biography

Academician Milosav Marjanović was born on August 24, 1931 in Krstac near Nikšić (Montenegro, The Kingdom of Yugoslavia), where his father Milan was appointed as a teacher.

He completed both his primary and secondary education in Nikšić. He received a diploma in mathematics from the Faculty of Natural Sciences, University of Belgrade in 1955 where he began to work as a teaching assistant in mathematics in 1957. He received his Ph.D. degree in mathematics from Belgrade University in 1964 with the thesis “Moore-Smith Convergence in General Topology” (supervisor Professor Djuro Kurepa). He spent his nearly entire professional career affiliated with the Department of Mathematics of the Faculty of Natural Sciences, University of Belgrade where he was appointed as an assistant professor in 1964, associate professor in 1969, and where he attained the position of a full professor in 1980. A few of the last years before retiring, he worked as a professor at the Teacher Training Faculty of the University of Belgrade.

He spent the 1959/60 school year at the Mathematical Institute of the Polish Academy of Sciences in Warsaw, joining the research seminar led by distinguished Polish mathematicians J. Mikusinski and R. Sikorski. During the 1967/68 school

year he had a visiting position at the University of Florida at Gainesville where he joined the research seminar of the outstanding Dutch topologist J. de Groot.

M. Marjanović was elected in the Serbian Academy of Sciences and Arts as a corresponding member in 1976 and as a full member in 1991.

Research in mathematics

Talking about the greatest achievements in the history of mathematics Prof. Marjanović, on one occasion, emphasized the creation and development of various “number systems”. However, his early fascination with an elusive mathematical concept of the “shape” of a mathematical object, led him to the study of topology.

His main results are related to the topologies on collections of closed subsets. He gave a complete classification of topological types of the spaces of closed subsets in the case of zero dimensional, compact, metrizable spaces. His “*beautiful theory of accumulation orders and spectra of compact metric zero dimensional spaces*” (words of S. Todorčević) led him to discover some subtle properties of these spaces, which allowed him to extend and complete some results of G. Choquet, V. Ponomarev, A. Pelczynski, P. Halmos, and others. This theory is arguably the most important and far reaching work of M. Marjanović, so here we briefly review some of the central concepts and fundamental results.

Given a zero dimensional compact metric space X , to each point $x \in X$ a natural number or “infinity”, $\text{Ord}(x) \in \mathbb{N} \cup \{\omega\}$ is inductively associated, called the *accumulation order* of x . The increasing sequence $s(X) := \{n \in \mathbb{N} \cup \{\omega\} \mid (\exists x \in X) \text{Ord}(x) = n\}$ is called the *accumulation spectrum* of X .

A space is called *full* if for each n the closure of the set $X_n := \{x \in X \mid \text{Ord}(x) = n\}$ is either empty or homeomorphic to the Cantor set C .

The first fundamental observation of the theory of accumulation orders is that two full spaces X and Y are homeomorphic if and only if $s(X) = s(Y)$. This result reduces the study of mono or poly functors \mathcal{F} on the category of full zero dimensional, compact, metrizable spaces to a more combinatorial (algebraic) problem of describing the accumulation spectrum of $\mathcal{F}(X)$ in terms of the accumulation spectrum of X .

[1] *Exponentially Complete Spaces III*, Publications de l’Institut Mathématique, tome 14 (28), 1972, pp. 97–109.

In the paper [1] this program was carried on for the “hyperspace functor” $\mathcal{F}(X) = \text{exp}(X)$, where the *hyperspace* $\text{exp}(X)$ is the space whose points are non-empty, closed subsets $F \subseteq X$, topologized by the Hausdorff metric (Vietoris topology).

These results allowed M. Marjanović to achieve a complete classification of “exponentially complete spaces”, satisfying $X \cong \text{exp}(X)$, extending the results of G. Choquet and A. Pelczynski, and to generate examples of non-homeomorphic, zero dimensional, metric, compact X and Y such that the corresponding hyperspaces are homeomorphic $\text{exp}(X) \cong \text{exp}(Y)$ (a problem of V. Ponomarev).

A similar strategy was applied to the product functor $\mathcal{F}(X, Y) = X \times Y$ in the paper

- [2] *Numerical invariants of 0-dimensional spaces and their Cartesian multiplication*, Publications de l'Institut Mathématique, tome 17 (31), 1974, pp. 113–120.

In this paper M. Marjanović addresses the problem of determining the spectrum $s(X \times Y)$ if the spectra $s(X)$ and $s(Y)$ are known. This problem is equivalent to the problem of describing the function $\mathcal{M}(m, n)$ such that for each $(x, y) \in X \times Y$,

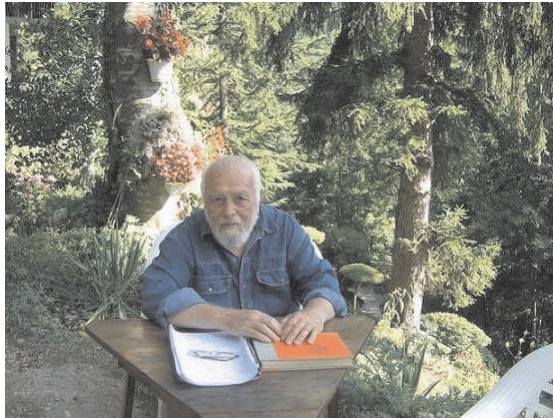
$$\text{Ord}(x, y) = \mathcal{M}(\text{Ord}(x), \text{Ord}(y)).$$

He discovered a remarkable formula

$$\mathcal{M}(6m_1 + r_1, 6m_2 + r_2) = 6(m_1 + m_2) + \mathcal{M}(r_1, r_2)$$

where for $r_1, r_2 \in \{0, 2, 3, 4, 5, 7\}$ the corresponding value $\mathcal{M}(r_1, r_2)$ is explicitly calculated. Among the consequences is the construction of non-homeomorphic, zero dimensional, compact metric spaces X and Y , such that $X^2 \cong Y^2$, which answered a well known problem posed by P. Halmos in his *Lectures on Boolean Algebras* (Van Nostrand, 1963).

A complete exposition of this theory can be found in the monograph S. Todorčević, *Topics in topology*, Springer, 1997, see also *The Teaching of Mathematics XIV-2 (2011)*, where a list of ten representative papers of Prof. Marjanović are presented.



Milo at his mountain retreat

Pedagogical work

As a university professor M. Marjanović substantially contributed to the mathematics education at all levels, from the elementary school to the university, by producing highly influential textbooks and research papers, and by having an important role in the innovation of school and university curricula.

He prepared, together with collaborators, several mathematical textbooks for pupils in elementary school (7–10 years old). His textbook (two volumes) “Methodology of the Teaching of Mathematics” (in Serbian), aimed for students of teacher

training faculties, was published in 1996. In his textbook “Mathematical Analysis I” (in Serbian), published in 1979, the basic concepts of analysis are introduced in the framework of metric spaces. He was the first to introduce elements of algebraic (combinatorial) topology into the curriculum of the Faculty of Mathematics in Belgrade. His textbook “Topology” (in Serbian) was published in 1990. and the updated version, written together with Siniša Vrećica, was published in 2011. His monograph “Structuring of number systems” (coauthor Zoran Kadelburg), printed in 2023, addresses didactic questions and problems relevant for students of mathematics and, in simplified form, teachers in elementary and primary schools.

He was for the last 32 years the editor-in-chief of the pedagogical journal *Nastava matematike*, which has been published by the *Mathematical Society of Serbia* (Društvo matematičara Srbije) since 1952.

According to the *Mathematics Genealogy Project* Milo Marjanović has 8 students and 15 descendants:

<https://www.genealogy.math.ndsu.nodak.edu/id.php?id=54002>,

however this is only a fraction of students and young mathematicians who benefited from his guidance, mentorship, support and friendly advice.

The Teaching of Mathematics

Milo Marjanović was from the very beginning (in 1998) the *spiritus movens* and one of editors-in-chief (together with V. Mičić, and later Z. Kadelburg) of the journal “The Teaching of Mathematics”,

<http://www.teaching.math.rs/index.php>,

also published by the Mathematical Society of Serbia. He has contributed 25 articles to this journal, more than half as a sole author.

Legacy of Milo Marjanović

Prof. Marjanović is highly regarded for his contribution to the foundation of pedagogy and didactic of mathematics in Serbia.

His beautiful and elegant results about hyperspaces, accumulation orders and spectra of compact metric zero dimensional spaces, reserve for him a permanent place in the edifice of the 20th century topology.

His pronouncement that combinatorial (algebraic) topology should come in front, to be developed and applied all over mathematics, is followed by his students and students of his students.

Milo Marjanović will be missed by his friends, students, colleagues, and everyone familiar with his work and his personality, but his legacy will live on for many years to come.

Rade Živaljević