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ANDREÍ NIKOLAEVICH KOLMOGOROV (1903–1987)
The great Russian scientist

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Abstract. The article is devoted to the hundredth anniversary of great Russian scientist A. N. Kolmogorov. He made contributions to several fields of science and nearly all parts of mathematics. His contributions to mathematics are described here, classified as: results, ideas, theories and methods and concepts. Some results from other natural sciences are also presented, as well as his continuous work on teaching of mathematics.

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Andreí Nikolaevich Kolmogorov undoubtedly was one of the greatest mathematicians and researchers of laws of nature of the Twentieth Century, (a Natural Philosopher, as such one would have been called in earlier times), and one among the greatest Russian scientists in the entire history of the Russian science.

About the magnitude of his scientific achievements one can judge by considering the wide international recognition of his work. The number of foreign Academies of Sciences that had him as a foreign member, and number of universities that granted to him the honorary degree, is surpassed only by Russian scientists I. P. Pavlov and P. L. Kapitsa. He was a member of the National Academy of Sciences, USA, the American Academy of Arts and Sciences, the French Academy of Sciences, the London Mathematical Society, the German Academy for Natural Sciences "Leopoldina", a member of the International Academy for History of Sciences, a member of the Netherlands Royal Academy of Sciences, and the Finnish Academy of Sciences. In addition, he was a member of national academies of sciences of Poland, Rumania, and Hungary, and of many other academies and scientific societies.

A. N. Kolmogorov was a recipient of the International award granted to him by the Bolzano Foundation, the award that was considered to be more meritorious than the Nobel prize. He was the first contemporary mathematician to whom this prize was awarded, other laureates being the Pope, and the three greatest representatives from the world of arts and sciences (historian Morrison, biologist Frisch, and composer Hindemith). In addition, he was a recipient of the international prize awarded to him by the Wolf Foundation.

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Within his homeland, Kolmogorov was honored with many high state awards, and academic awards bearing names of Chebishev and Lobachevskií. There is no doubt about it, that for studies of turbulence he would have been awarded the Nobel prize, had he lived a little longer.

Andreí Nikolaevich was the founder of the great scientific school. Fifteen of his best students became either full members, or associate members of the Russian Academy of Sciences.

Among his students, there are about sixty university professors, while several dozens among them became leaders in many scientific disciplines. I mentioned the Wolf Prize whose recipients are about fifty greatest mathematicians of the second half of the Twentieth Century. Among these recipients there are six who are our contemporaries (besides Kolmogorov, Gel'fand, Mark Kreín, Sinaí, Arnol'd, and Piatecki-Shapiro). Three of them, Arnol'd, Gel'fand, and Sinaí, are Kolmogorov's pupils. His scientific contributions and the establishment of the scientific school are deeds benefiting the whole humankind.

Andreí Nikolaevich performed a great service to his country. He was a reformer of the university and high school mathematical education in Russia, he served at many governmental posts dealing with mathematical education, and he served for many years as the President of the Moscow Mathematical Society. Kolmogorov was an enlightener, and he was the founder of Moscow's, Interstates, and all-Russia's mathematical olympiads. He was the founder of *The Boarding School*, which was renamed after him since 1980. He was among founders of many scientific journals such as *The Survey of Mathematical Sciences*, and of the first specialized scientific journal, *Theory of Probability and its Applications.* In addition, he was the founder of the youth journal *The Quantum*, and the editor of the accompanying book series *Quanta.* Finally, he wrote the entire part about mathematics for both, the first, and the second Great Soviet Encyclopedia.

For more than sixty years A. N. Kolmogorov taught at the Moscow University. During the period between 1954 and 1958 he served as Dean of the College for Mechanics and Mathematics; at this college, he was the founder and chair of Department for Probability Theory, and its associated laboratory for probabilistic and statistical methods. He was head of Department for Mathematical Logic, and he was one of the founders of Department for Mathematical Statistics and Theory of Stochastic Processes. He passed all this to his pupils.

Kolmogorov's scientific image is characterized by his unsurpassed universality. Perhaps the best description of this remarkable feature of Kolmogorov was given by his friend, the well-known topologist and academician, Pavel Sergeevich Aleksandrov: "the unusual breadth of creative interests of A. N. Kolmogorov, coupled with a wide span and variety of the areas of mathematics in which he has worked in various periods of his life, distinguish him within mathematicians, not only in our country, but world-wide. It could have been said rightfully that his mathematical talent was unmatched among mathematicians of our time."

It is common for a scientist to have one narrow field of specialization, say, Theory of Probability, Differential Equations, Algebraic Geometry, etc. It is less common for a scientist to have two such fields; but, it is quite uncommon for a scientist to possess three such fields. However, in the case of Kolmogorov, one finds more than twenty narrow areas of mathematics which he either started, or, in which he became their classical contributor. Here is the list of such areas: Theory of Functions in Metric Spaces, Descriptive Set Theory, Mathematical Logic, Classical Theory of Probability, Geometry, Theory of Stochastic Processes, Mathematical Statistics, Functional Analysis, Theory of Approximations, Topology, Differential Equations, Theory of Turbulence, Theory of Ballistics, Theory of Algorithms and Automata, Classical Mechanics, Theory of Superpositions of Functions, Theory of Information, Theory of Dynamical Systems, and Theory of Complexity and Algorithmic Theory of Chaos.

Besides in mathematics, he has made many important contributions in the following sciences, some of them being close to mathematics, and some quite remote from it: Mechanics, Physics, Biology, Geology, Oceanography, Meteorology, Crystallography; in humanities: History of Russia, Linguistics, Poetry, and History of Science.

In the creative process of any outstanding mathematician one may discern the presence of the following components: his *results*, (solutions of problems and proofs of difficult theorems); his *ideas*, (introduction of new concepts, re-formulation of old concepts, posing of new problems, starting new scientific directions); his *theories*, (when many of results are synthesized to form a new, complete body of knowledge); his *methods and concepts*, (where his main ideas, not only serve mathematics, but acquire deep philosophical meaning as well). However, only in creative works of the greatest minds such as, say, that of Poincaré, one finds all of these components present. Also in the case of Andreí Nikolaevich Kolmogorov. However, in his creative process there was an additional, the most important characteristic present - the closest connection with problems of natural sciences.

A review of the whole opus of Kolmogorov is impossible to present in one journal article. Therefore, I am going to continue with review in this "catalog style", having in mind that to do a justice to this presentation it would require the space of two to three tomes. Let us start with this "catalog".

Results.

• An example of almost everywhere divergent Fourier series (this example was constructed in 1922)—it is a corner-stone in the Theory of Trigonometric Series.

• Weak Hilbert Transform (1925)—a basic result in Harmonic Analysis.

• Necessary and Sufficient conditions for the Strong Law of Large Numbers—one of many results obtained by Kolmogorov that are completing the development of the classical probability theory (1928).

• Characterization of infinitely divisible laws (1931)—derivation of one of the basic formulae of the Theory of Probability (the so-called Levy-Hinchin formula; Levy derived the formula by relaxing Kolmogorov's assumption about finite variance, while Hinchin demonstrated how by following Kolmogorov's argument, one can obtain the general result).

• The limit distribution of the difference between the theoretical distribution (unknown), and the sample distribution (1933)—this is one of the basic theorems of Mathematical Statistics.

• Construction of an open mapping that increases dimension (1937)—a classical result in General Topology.

• An estimate of the error of approximation of the distribution of the sum of independent summands by infinitely divisible distribution (1956)—this was the last outstanding Kolmogorov's result in the Theory of Probability (today it is known as Kolmogorov-Arac theorem).

• Representation of a continuous function of several variables on a cube as a superposition and sum of functions of a single variable (1957)—the final result concerning a Hilbert's problem.

• Proof that Bernoulli shifts with different ps are not isomorphic (1958)—this solves a problem posed by John von Neumann.

Ideas

Their number contain many works dealing with unifying ideas of basic mathematical concepts such as integral, derivative, divergent series; definition of the concept of k-dimensional measure of a set contained in n-dimensional space; the final results of the theory concerning the concept of δs-operations, whose origin one finds in works of P. S. Aleksandrov and M. Ya. Suslin, and whose further developments one finds in memoirs of L. V. Kantorovich (who considered Kolmogorov one of his teachers), E. M. Levinson, and A. A. Lyapunov; interpretation of the classical mathematics from a standpoint of the intuitionism; taking into the consideration at the same time, the logic of proofs, and the logic of "problem solving"; his work in "Topological Geometry" which is the starting point of Topological Algebra; presenting definition of a linear topological space and proving the criterion for making a decision whether such a space could be made into a normed space; introduction of two new directions in Approximation Theory (approximation by a class of elements using a given approximation procedure, and finding the optimal procedure for approximation among the class of such procedures); a basic result (obtained jointly with I. M. Gel'fand), in Theory of Banach Algebras; definition of the co-homology of complexes; the most general definition of the concept of an algorithm; definition of the entropy of a dynamical system, and ε-entropy of a class of functionals—all these are but a few of ideas, concepts, and new directions, brought into mathematics by Andreí Nikolaevich Kolmogorov.

Theories

One of the most important works of Andreí Nikolaevich Kolmogorov is his book "The Basic Concepts of Probability Theory" (1933). By this work, by formulating its system of axioms, Kolmogorov has elevated Probability Theory to the level of a mathematical science. For a reason, this work is compared by its importance with fundamental memoirs on the subject written by J. Bernoulli (who proved for the first time the Law of Large Numbers Theorem), and that of Laplace (who proved the first version of the Central Limit Theorem of Probability Theory).

A very important contribution A. N. Kolmogorov has made in development of Stochastic Processes. In 1931 he has published his outstanding work "Analytical Methods in Probability Theory". In an article dedicated to the semi-centennial of A. N. Kolmogorov, P. S. Aleksandrov and A. J. Hinchin wrote: "In the whole history of Probability Theory in the Twentieth Century, this work of Andreí Nikolaevich gave the foundation for further development of Probability Theory and its applications". The mentioned Kolmogorov's work is remarkable by the simplicity and depth of its pioneering methodology. Investigation of a deterministic process where the initial state of the process in the phase space determines the further evolution of the process, was extended to investigation of stochastic processes in which, according to the authors of the article, "state x of a process in some instant t_0 conditions the probability that process will be in state y of the phase space at some future instant $t > t_0$". This observation led A. N. Kolmogorov to definition of a Markov Process. In this instance, he derived in general case an integral equation that was earlier obtained for particular cases by M. Smoluchowski. Using this integral equation, he derived the so-called *forward equation* (already used in papers of the greatest physicists Planck, Einstein, Fokker, and Smoluchowski), and the so-called *backward equation*, that was not known to physicists. This monograph of Kolmogorov presents the synthesis of the Heat Theory as started by Fourier, Theory of Brownian Motion of Einstein-Smoluchowski, description of Markov Random Walk, as well as of basic ideas of Bachelier and Wiener—the last two being ones who constructed the first models of stochastic processes.

In mid thirties of the Twentieth Century, A. N. Kolmogorov and A. Ya. Hinchin have developed the Theory of Stationary Stochastic Processes (in 1941 for this work they have been awarded the State Prize).

In monograph "Limit Distributions for Sums of Independent Random Variables", which was written jointly with B. V. Gnedenko, authors have compiled results of the classical period of the research on this subject encompassing two and a half centuries of studies of sums of independent random variables starting with works of J. Bernoulli, A. Moivre, P. Laplace, and continued in works of S. Poisson, P. L. Chebishev, A. A. Markov, S. N. Bernstein, and others. In 1951, for their research in the area of sums of independent random variables, authors of the mentioned monograph were awarded P. L. Chebishev Prize.

In 1965, for their work in the area of Classical Mechanics which is the foundation of the so-called "KAM-Theory", (Theory of Kolmogorov-Arnol'd-Mozer), Kolmogorov and Arnol'd got awarded the highest prize at that time for scientific research.

In 1986, for his theory of co-homology and co-homological operations, Kolmogorov was awarded N. I. Lobachevskií prize.

Andreí Nikolaevich has made a great contribution in the area that was once called Philosophy of Nature, or, Natural Philosophy, i.e., in Natural Sciences. Vladimir Igorevich Arnol'd, one of the great mathematicians of our time, once has said: "Kolmogorov – Poincaré – Gauss – Euler – Newton, are only five lives separating us from the source of our sciences". Here are life spans of these great scientists: Newton (1643–1727), Euler (1709–1783), Gauss (1777–1855), Poncaré (1854–1912), Kolmogorov (1903–1987). We see that a life span of each of these individuals, even for a short period, has overlapped with the life span of his predecessor. In fact, we have here a continued life thread. Five lives—however short their total life span, but how boundless their accomplishments! Newton, Euler, Gauss, and Poincaré, are giant landmarks on the road of development of Natural Sciences; and Arnol'd includes Andreí Nikolaevich Kolmogorov among these giant landmarks.

One of the central questions in Natural Sciences is that about the structure of the Solar System (and its possible origin). Newton was the first one who has formulated laws of the planetary motion and to have posed the question about the structure of planetary systems. Euler, Gauss, and Poincaré, as well as many, many other scientists have worked to find answer to that question. The question of questions in Theoretical Astronomy, as well as one of fundamental questions of Natural Sciences is: can a system, like the Solar System, exist forever? Or, is it inevitable (on a set of the complete measure of the initial conditions, that is, "almost forever") that the evolution of similar systems leads to catastrophe? In 1953 it was destined that Andreí Nikolaevich breaks the deadlock of this great problem.

The spiritual uplift that ensued then in the life of Kolmogorov, led him to formulation of the new method for proving theorems of existence, that (with the addition of results of V. I. Arnol'd and J. Mozer) is known today under the name of KAM-Theory (Theory of Kolmogorov-Arnol'd-Mozer).

Creation of KAM Theory brought about real advances for treatment of this set of questions. Problem of describing the motion of systems which are close to integrable systems of Classical Mechanics, dates back to Newton. Laplace has made the first steps towards solution of problems of stability for similar system by using Theory of Perturbation. Commenting about works of A. N. Kolmogorov, V. I. Arnol'd writes that Poincaré, "after he has analyzed numerous attempts of substantiation and refinement in arguments of Laplace, has given to the problem the modern formulation [...] and has called it the basic problem of dynamics [...] Mentioned works of Kolmogorov give the solution of this problem in a case with general position, and for the most cases with given initial conditions". During various years, Kolmogorov has had repeatedly fruitful contacts with physicists (S. I. Vavilov, M. A. Leontovich, and with others). A paper, "Calculation of the Mean of Brownian Surface", written jointly with M. A. Leontovich, solved the problem posed by S. I. Vavilov. Andreí Nikolaevich loved to mention that to him, as to a mathematician, belongs the physics part of the paper, while to physicist Leontovich belongs the mathematics part of the paper.

His intuition for grasping physical phenomena, his clear understanding of the physical nature of processes of undamped oscillations with continuous spectra, has served Andreí Nikolaevich as a trail blazing star to formulate (jointly with A. Ya. Hinchin), the Theory of Stationary Random Processes. Stationary Random Processes, that is, Random Processes whose probabilistic characteristics are invariant with respect to time, serve as models for large number of stochastic phenomena in nature (in Earth's atmosphere, in Oceans, etc.). These processes are encountered all the time in direct technical applications. A problem of the greatest interest in both, natural sciences and engineering, is that of prediction of behavior of a process based on observations of that process over some time set. Prediction Theory for stationary random processes was developed by Kolmogorov and N. Wiener. Wiener has considered this theory to be one of his greatest achievements, and somewhat with a feeling of bitterness has acknowledged Kolmogorov's priority for its development.

Now, about Kolmogorov's research concerning the problem of turbulence. His work in this area has been developed on the basis of the phenomenological research of the greatest scientists-mechanists of the past Century, Geoffrey Taylor and Theodore von Kármán. Kolmogorov's contributions have completely changed then existing theory of turbulence, and have made a major impact on the whole development of this area of natural sciences. Description of this impact one can find in the monograph of the well-known French scientist U. Frisch published in the year 1998. "Turbulence. The Heritage of Kolmogorov"—is the title that Frisch gave to his trail blazing book. In it, one finds such words as: "is it by chance that deeper than anyone else into the essence of turbulence has penetrated Kolmogorov—a mathematician, possessing unsatiable interest for the living reality?" References cited in Frisch's book contain more than six hundred citations of the followers of Kolmogorov's work. However, these citations are only a small part of existing works that have sprung from Kolmogorov's theory. Among followers of Kolmogorov in this area are distinguished scientists in the applied fields of sciences, such as academicians Andreí Sergeevich Manin and Aleksandr Mihailovich Obuhov.

Very important research contributions made by Kolmogorov in natural sciences have been inspired by themes from biology. In the year 1932 director of the Institute for Experimental Biology, outstanding Soviet biologist N. K. Kol'cov has formed in his Institute a not-too-big evolution "brigade" (or, as we would call it today, a laboratory for studying problems of evolution). The Lab's head was Dmitrií Dmitrievich Romashov, Kolmogorov's high school classmate. Andreií Nikolaevich took an active participation in Lab's activities. He advised D. D. Romashov to bring as Lab's consultants V. I. Glivenko, A. A. Lyapunov, and N. V. Smirnov, while he himself has established fruitful scientific contacts with geneticists A. S. Serebrovski, N. P. Dubinin, A. A. Malinovski, as well as with many other biologists. These pursuits led Kolmogorov to solutions of numerous problems in Mathematical Biology. In the year 1937, A. N. Kolmogorov, I. G. Petrovskií, and N. S. Piskunov, have published a joint monograph entitled "Research on Diffusion Equation Associated with the Growth of Substance and its Application to a Problem in Biology". This work has achieved an unusually high level of development in various problems in natural sciences (in particular, in the theory of fire spreading originated in works of J. B. Zeldovich and D. A. Frank-Kamenec, in the theory of burning and explosions, in the theory of neuron cells impulse transfers, etc.). It is interesting to observe, that here as in his previous work with Leontovich, Kolmogorov gets credit for description of the underlying physical model, while its mathematical model was essentially developed by his co-authors. Kolmogorov commented that the observed physical experiment that was needed for building of the biological model, was the spread of flame while a special cord was burning ("I have seen the cord burning!" he said, and after leaving this visual experience, he thought off the model of the phenomenon described by a non-linear differential equation whose solution propagates with a constant speed, while preserving the form).

In the year 1940, Kolmogorov published his monograph entitled "A New Confirmation of Laws of Mendel". During that year a discussion has unfolded between geneticists and the followers of Lisenko, concerning the validity of Mendel's Laws. In order to settle the dispute, Lisenko and N. M. Vavilov have asked their collaborators, N. I. Ermolaev, and T. K. Enin, respectively, to replicate Mendel's experiments in order to either disprove (this job was given to Ermolaev), or to prove (the job was given to Enin), Mendel's Theory. Both investigators "coped well" with their assignments. Then, Andreí Nikolaevich has studied resolutely results obtained by both investigators. In the introduction of the mentioned monograph, he states: "Not only that Mendel's theory leads to the simplest conclusion about the stable approximation of the ratio's value 3:1, but gives also a way for predicting the mean deviation from this sample ratio. Based on this statistical analysis of deviations from the ratio 3:1, one gets a new, sharper and more exhaustive method for verification of Mendel's statement about splintering of heredity signs. The goal of the present note is to point out the most rational method for testing and illustrations, which according to the opinion of the author, (ANK), is contained in data provided by N. I. Ermolaev. These data, contrary to what N. M. Ermolaev wishes, provide a new, bright confirmation of Mendel's Laws." (Andreí Nikolaevich said that Ermolaev conducted his experiments and data gathering in a remarkable conscientious manner, which enabled him to provide a new, bright confirmation of Mendel's Laws, while Enin, who right away wanted to confirm the Laws, obviously rejected some results of experiments, (which didn't look favorable towards this goal), and by so doing have obtained the better results. About this fudging of Enin, Andreí Nikolaevich states in the monograph: "attaching systematically excessive approximation of frequency m/n to $3/4$, is what one finds in data of Enin".)

A step from dealing with problems in biology led Kolmogorov to develop the theory of Branching Processes. A pupil of Kolmogorov, one of the leading experts in theory of Branching Processes, B. A. Sevastianov, writes: "the general concept as well as the term "Branching Random Process", which became immediately widely accepted, have been publicly announced by Kolmogorov at his seminar at the Moscow State University during 1946/47 school year. Many problems connected with simple models of Branching Processes have been considered earlier by many people. In particular, one such a problem was solved by Kolmogorov in one of his earlier papers. ["Towards Solution of a Biological Problem" (1938)]. At the present time, there are numerous monographs dealing with Branching Processes [...] Kolmogorov's model of Branching Processes turned out to be very effective, both, for producing directly new results, and for its applicability for solving problems in biology, chemistry, physics, and engineering."

Among works included by Andreí Nikolaevich in the survey of his selected papers, there are many others dealing with problems in natural sciences, for example, papers in crystallography and geology (He has selected these papers for the survey, thus pointing out to their importance).

Kolmogorov's contributions to natural sciences represent one of the highest peaks of achievement of science in the Twentieth Century.

The conception of Chaos crowns Kolmogorov's creative contributions. By its algorithmic complexity, it represents one of the greatest philosophical achievements of the Twentieth Century.

The whole life of Andreí Nikolaevich has been dedicated to teaching. He taught in a regular high school (three years in his youth, 1922–1925, then, he taught in an experimental school, both, mathematics and physics); he taught a lot in the Boarding School # 18, which he helped establishing; he introduced teaching of mathematics in the Vocational-Pedagogical Institute Liebknecht. Finally, he taught sixty years as professor in the College of Mechanics and Mathematics at the Moscow University. During this time, he taught large number of required and specialized courses, and he introduced applied problem working sessions in analysis and theory of probability.

As an example, let me tell you a story about a special course that Andreí Nikolaevich entitled "Analysis III". Structure of that course reflected Kolmogorov's idea about the unity of mathematics, in particular, about a deep connection between the pure and the applied mathematics, and of necessity (as stated in introduction of the second edition of the book) "to educate students in acquiring a dual point of view: on one hand, to follow the inner logic of set theory, of the theory of continuous maps in metric and general topological spaces, of linear spaces and functionals, of operators defined on them, of abstract measure theory, and of integration in general "measure spaces", while on the other hand, without losing the insight, by using these abstract areas of mathematics, to deal with problems of classical and applied analysis". Andreí Nikolaevich promoted the idea about necessity for "synthetic" university courses that will reflect the unity of ideas acquired by students in their early stages of education on elementary levels. In the mentioned course Analysis III, such a synthesis has been achieved including materials from Analysis I, and from Differential Equations, (Analysis II). All these topics have been integrated into a course that contains topics (found in earlier textbooks), from the Theory of Real Functions, the Functional Analysis, the Calculus of Variations, the Integral Equations, as well as some other topics.

As a consequence of this endeavor, similar synthetic courses have been developed in two very important areas of mathematical education—Algebra and Geometry (courses such as "Linear Algebra and Geometry", and "Differential Geometry and Topology".)

Active to these days programs for teaching mathematics have been designed under the leadership of Kolmogorov. In their time, these programs were very modern and they were characterized by a wide grasp and expediency.

The last years of his life, Andreí Nikolaevich Kolmogorov has dedicated to high school mathematics education. He thought of a new plan, in particular, as how to transform teaching of the high school course in Geometry. I have learnt about this concept of his, (actually, I was told about it), only after Andreí Nikolaevich has passed away. According to my understanding, he had the idea to describe a Euclidean plane in two ways: visually and rigorously. For example, a visual description of the plane one can find in high school books written by Kiselev, while the rigorous description of the plane does not exist in high school books. In Kolmogorov's visual characterization of the plane one can find a synthesis of the old Euclidean way of looking at the plane (a part which could be imagined as a school board on which one can perform constructions using a ruler and a compass—such as to construct lines passing through two different points, to measure distances between points, to construct a circle of a given radius, in other words, to do all this which is usually done in schools), and the concept of Geometry that was introduced by Felix Klein.

Visual Euclidean description of the plane, Kolmogorov has supplemented with two new vantage points, by following, on one hand, the concept of distance (which has been axiomatically introduced by M. Frechet in the year 1912), and on the other hand, the Erlangen Program of Felix Klein (1872). According to the Erlangen Program, the Euclidean plane is characterized by the group of motions (that is, by a set of isometric transformations).

One can understand easily and visually the essence of motions by adding a transparent piece of glass to a school board. If on the board there is a drawing of some figure, (say, of a triangle), this drawing could be copied on the glass, the glass could be moved to a new location on the board, and the drawing copied back from the glass to the board. This way an isometric (distance preserving) transformation of the drawing has been obtained. The piece of glass could be "flipped", leading to the drawing symmetric to the original one. As examples of motions, one observes translations, rotations with respect to a fixed point in the plane, and symmetries with respect to a line.

By adding to Euclidean Geometry concepts of metric and motion, it becomes possible using the first concept, to give beautiful proofs of many theorems, while using the second concept, to solve various problems in Geometry.

Take for example, one of the first theorems in Geometry (attributed to Thales), which states that two angles corresponding to the congruent sides of an isosceles triangle are congruent. Here we can use the concept of motion, (so to speak "using glass"), to prove this theorem: copy the triangle from the board onto the glass, then flip the glass, and put again the copied image on the glass over the triangle on the board. Two triangles, the original one on the board, and the flipped one on the glass will overlap, thus showing that angles in question are congruent (they have changed positions).

All that was used here follows from the visual description of the plane. Rigorous description of the plane is possible by introduction of coordinates and the ensuing use of algebraic methods. Such a description was proposed by Hermann Weyl. However, it is possible to have a geometric-axiomatic rigorous description of the plane. Such an approach has started by Euclid and was completed by the great mathematician David Hilbert (1862–1943). Hilbert's life span straddled the Nineteenth and the Twentieth Century.

Kolmogorov follows the geometric approach, too. However, Kolmogorov's axiomatic system is unlike that of Euclid-Hilbert's. Nevertheless, its axiomatic system is simple and natural. Like in Euclid's system, he puts into axioms visual properties of lines, but then, he adds axioms describing distances, segments, half-spaces, as well as motions (for example, he postulates that two different points determine one and only one line, that distance is symmetric, and that it satisfies the triangle inequality, that there are exactly two motions that take a given segment onto another one of the equal length). Finally, parallel postulate was added, stating that through a given point laying outside a given line in a plane, there is only one line parallel to a given line. Moreover, if the existence of more than one parallel line is postulated, one gets another Geometry, such as that of Lobachevskií.

Based on some of his statements, Andreí Nikolaevich dreamed that teachers who love their subject, are able during special sessions and additional workshops, to introduce to highly motivated students, the world with different geometries, especially the finitely, and infinitely dimensional Euclidean world, the world with Lobachevskií geometry, the convex world of Minkowski, affine and projective worlds ... It has to be honestly admitted, that such magnificent ideas were unsuitable for acceptance into mass schools. (And not only they would not be accepted for years to come, but, perhaps, forever).

With a chagrin, I have learnt about all these, slightly late. I felt bitter on one hand, that I didn't try to dissuade Andreí Nikolaevich in his efforts for realization of his dream, while on the other hand, I was pleased to have learnt about beautiful ideas of my teacher, after he was gone. I feel sorry that I did not come to his defense when he needed it.

Kolmogorov's recommendations for reforms of the high school mathematics program are still awaiting a careful analysis and an objective evaluation.

A. N. Kolmogorov was a genius and a person proficient in a wide range of fields. He was interested in sciences, for both, exact ones, and humanities, and he had a keen interest for philosophical problems as well as for problems of ethics and morality. He was an expert and a delicate judge of arts—of poetry, of paintings, and above all, of sculptures. He was deeply concerned for the future problems of humankind.

A special trait of Kolmogorov's creativity was his ability to concentrate his enormous intellectual capability over a relatively short period of time. When he was talking about his construction of an almost everywhere divergent Fourier series, he recalled three time periods of 24 hours duration each when with uninterrupted thinking and a complete concentration, he suddenly arrived at desired example.

Similar concentration would have caused a powerful explosion—it appears into an impregnable fortress, thus pointing out instantaneously the light at the end of the tunnel, towards which tens, if not hundreds of researchers, would have be rushing. However, Andreí Nikolaevich usually did not follow such a flow—he would have experienced a creative relaxation, and in his thoughts he would have headed towards new pursuits.

Kolmogorov belongs to a group of "fast geniuses", (the sprinters, according to Arnol'd), who could acquire new information very fast, and whose brain was capable of unifying ideas, seemingly ununifiable.

Kolmogorov was interested in studies of gifted creativity. He distinguished three types of mathematical creativity: algorithmic, visual-geometric, and logical. Today, we classify them into two categories, (analytic, and form-geometric), corresponding to our present understanding of brain's hemispheres's functions. Andreí Nikolaevich was a great analytical mind, and a great geometer. Often, he would tell that discoveries have appeared to him in geometric forms (although, his discovery of an almost everywhere divergent Fourier series, is a pure analytic fact).

Kolmogorov loved sporting activities. During his leisure time, he took long walks, and he was an excellent swimmer. He loved to travel - by himself, with friends, with pupils. Those who knew him well, remember him as a good, touching person, often with a warm, and radiant smile on his face. In earnest, he seldom was angry.

He could empathize with a person in difficulties, and to provide a real help where needed, and he gave his support to people when needed.

In all, Andreí Nikolaevich was without jealousy and envy towards accomplishments of others. He never displayed an air of arrogance in dealing with individuals whose accomplishments he valued (even towards those who have treated him unfairly).

A spoken word of Andreí Nikolaevich was uniquely original. Once he wrote, that "he continues to have a problem with precise expression of his thoughts". Many of his aphorisms are strikingly unusual. Here are some of them.

"There is something in common to all enthusiasts of modest accomplishments—a desire to explain patiently their accomplishments to ones who do not care to understand these accomplishments."

"I want to underline here the righteousness and dignity of a mathematician who understands the role he plays in his science, in development of natural science and engineering, as well as in the whole culture of humankind, while he continues calmly to develop the "pure mathematics" in accordance with its inner logic of development."

"It is quite common in science (as well as in poetry, music, etc.), that a person [. . .] with all his right moral characteristics, perceives his work as a payment of an especially important debt."

For all times to come, Andreí Nikolaevich will serve as an example of enormous number of things that could be accomplished by a human life.