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WHAT FACTORS MAY INFLUENCE COLLABORATIVE PROBLEM SOLVING PERFORMANCE?

An eleventh grade study on solving a problem in several ways

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Abstract. By using a sample of 21 pairs of eleventh grade students (10 comprised mathematically-talented students), this study examined paired problem solving performance in terms of paired students' features concerning mathematical self-concept and cognitive empathy and found that collaborative problem solving performance was positively influenced by average mathematical self-concept for paired talented students. The talented pairs' bootstrapped data evidenced that this performance could be explained by a multiple liner regression model, where average mathematical self-concept for paired students and average cognitive empathy for paired students had zero or positive influence, whereas absolute mathematical self-concept distance for paired students and absolute cognitive empathy distance for paired students had zero or negative effect. Though not supported, the validity of that model was indicated by the average pairs' bootstrapped data.

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Introduction

According to [14], mathematical problem solving performance results from a complex interplay among solver's cognitive, metacognitive and affective domains, the last of which determines the global context where cognition takes place monitored and controlled by metacognition.

Attitudes toward mathematics are positively related to mathematics achievement (e.g., [5, 7]). According to [15], this relation holds true for mathematical self-concept operationalized in this TIMSS study by the statement "I usually do well in mathematics." A psychometrically tested measure of this construct, viewed as an organized system of beliefs supplemented by behavioral and emotional reactions regarding the value of mathematics and mathematical way of thinking as well as confidence in and motives for learning mathematics, was positively related to the mark (grade) in mathematics for the fall semester in each of four examined countries [8]. Having in mind the complexity of affective domain (attitudes seem to develop out of emotional responses; emotions usually occur when beliefs contradict

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the encountered situation; and attitudes are based upon beliefs; see [11]) as well as the fact that despite diversity affective variables regarding personality seem to converge to a unique pattern [12], such a mathematical self-concept may be a good affective domain representative and thus an important factor positively influencing problem solving performance.

To promote student learning, grouping (forming students into groups) seems generally (slightly) better approach than no grouping [16]. As regards mathematics in particular, success in solving problems in pairs may primarily be result of improved control behavior [3]. This behavior is influenced by perspective taking (higher-levels of perspective taking results in higher-levels of group discussion, see [6]), which may basically be shaped by cognitive empathy (see [2]). It thus seems that cognitive empathy may, among other metacognitively-related constructs, be an important factor positively influencing collaborative problem solving performance.

This study examined the influence of mathematical self-concept and cognitive empathy on paired problem solving performance. The main research question was: "Can paired problem solving performance be predicted by paired students' features concerning mathematical self-concept and cognitive empathy?".

Method

The study used a sample of 21 pairs of students who came from two eleventh-grade classes. Ten pairs were from *Matematička Gimnazija* (the specialized high school in Belgrade for mathematically talented students) comprising particularly talented students who participated in mathematical competitions. Other pairs were from *Prva beogradska gimnazija* (the oldest of 15 such high-schools in Belgrade) who can mostly be regarded as average mathematics students concerning the population of eleventh graders in Serbia.

The study had a correlative design. The variables were: cooperative problem solving performance (CPSP), average mathematical self-concept for paired students (MSC_{avr}), absolute mathematical self-concept distance for paired students (MSC_{dis}), average cognitive empathy for paired students (CE_{avr}), and absolute cognitive empathy distance for paired students (CE_{dis}).

CPSP was measured by the number of correct solutions to the following problem taken from [1]: "In the square below, M and N are midpoints of the corresponding sides. Determine the numerical value of $\sin\alpha$."

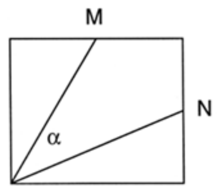

Fig. 1

The pairs, formed by the subjects themselves, generated and wrote solutions to this problem under a group setting (one class at a time; both classes within 40 minutes). The pairs' answers were scored by the author of this report. For each correct solution, pair received 1 point. Partial credit of .5 point was given for 4 out of 63 accepted solutions (these four solutions were almost correctly completed or fully completed with somewhat unclear/partially-wrong calculations).

Mathematical self-concept (MSC) was measured by a 15-item Likert scale utilized in [8]. The subject's mathematical self-concept was represented by the first principal component factor score obtained from the subjects' answers transformed into the Guttman image form[1]. According to the measure of Momirović and Knežević (see [9]), the reliability of such a factor score was at least .84. The values of variables MSC_{avr} (average mathematical self-concept for paired students), and MSC_{dis} (absolute MSC distance for paired students) were respectively determined by $\dfrac{MSC_1 + MSC_2}{2}$ and $|MSC_1 - MSC_2|$ where subscripts $_1$ and $_2$ refer to MSC value of particular paired student.

Cognitive empathy (CE) was measured by a 7-item Likert scale developed by Davis ([2]) available, for example, at `http://mailer.fsu.edu/~cfigley/Tests/IRI.RTF`. The instrument was translated into the Serbian language by the author of this study. The subject's cognitive empathy was represented by the first principal component factor score obtained from the subjects' answers transformed into the Guttman image form. The reliability of such a factor score was at least .81 (Momirović-Knežević). The values of variables CE_{avr} (average cognitive empathy for paired students), and CE_{dis} (absolute CE distance for paired students) were respectively determined by $\dfrac{CE_1 + CE_2}{2}$ and $|CE_1 - CE_2|$ where subscripts $_1$ and $_2$ refer to CE value of particular paired student.

The data collection was realized at the end of the spring semester in June 2005 during regular school lessons[2]. The author told the subjects that this study would examine their problem solving performance and the subjects willingly provided the requested data and eagerly solved the given problem in pairs.

To avoid possible limitations of drawing conclusions from quite small samples, the undertaken statistical analysis made use of bootstrapping since both types of pairs (talented students and average ones) could be regarded as good approximations of their corresponding populations. This statistical method, based upon resampling with replacement of the original data to generate a collection of new pseudoreplicate samples enabling confident inferences about the population (see [4]; see also `www.uvm.edu/~dhowell/StatPages/Resampling/Resampling.html`), was applied by means of the Provalis Research SimStat software (see `www.simstat.com`).

[1]This transformation, which eliminates noise from the initial data, is defined by

$$T = Z(I - R^{-1}U^2),$$

where T, Z, I, R and U^2 are, respectively, the matrix of true results, the matrix of the standardized (and perhaps normalized) initial data, the identity matrix, the matrix of the intercorrelation among the measured variables, and the matrix of an variance measurement error estimate given by $(\operatorname{diag} R^{-1})^{-1}$. The transformation can, for example, be realized by utilizing a SPSS syntax file written in the SPSS's macro language (see [7]).

[2]Eighteen talented students completed the two instruments assessing MSC and CE on Thursday and solved the problem in several ways on Friday (two students came on Friday and completed the instruments after accomplishing the task). On the next Friday the average students completed the instruments first and then immediately solved the task in several ways. The same patterns emerged when these two students (one pair) were excluded from the undertaken analyses.

Results

The means and standard deviations of the utilized variables for the type of pairs are reported in Table 1. The talented pairs solved the given problem in more ways (for equal variances, $t = 6.05$, $df = 19$, $p < .01$; for unequal variances, $t = 5.81$, $df = 10.95$, $p < .01$), demonstrating a higher variance of CPSP (5.11 vs. .61; Levene's Test: $F = 18.62$, $p < .01$). The examination of the group means in respect of variables 2–5 revealed no significant differences between the groups (talented vs. average).

Table 1. Means (standard deviations) of the measured variables for the type of pairs

VARIABLE	TALENTED	AVERAGE
1. CPSP	5.20 (2.26)	.82 (.78)
2. MSC_{avr}	.30 (.64)	−.28 (.89)
3. MSC_{dis}	1.12 (.73)	.83 (.60)
4. CE_{avr}	−.00 (.74)	.00 (.67)
5. CE_{dis}	1.15 (.91)	.91 (1.18)

The correlations among the utilized variables for the pairs of the talented subjects ($N = 10$) are presented in Table 2. One of these Pearson's correlation coefficients was significant at a .05 level. A multiple, stepwise type ($p_{in} = .05$, $p_{out} = .055$), linear regression analysis with CPSP as independent variable and MSC_{avr}, MSC_{dis}, CE_{avr} and CE_{dis} as dependent variables revealed that CPSP is only influenced by MSC_{avr}, which accounted for 46% (adjusted R square) of the CPSP variance ($F_{1,8} = 8.60$, $p < .05$).

Table 2. Correlations among the utilized variables for the pairs of the talented subjects

VARIABLE	2	3	4	5
1. CPSP	.72*	−.15	.25	−.20
2. MSC_{avr}		−.23	.47	−.13
3. MSC_{dis}			.10	−.58
4. CE_{avr}				−.16
5. CE_{dis}				

$^{*}p < .05$

For the talented pairs' data, a bootstrapping, two-variable analysis, based upon 1000 resamplings with 10 observations, was performed. It evidenced that only CPSP and MSC_{avr} were correlated (the 95% bootstrap confidence interval was $[.25, .92]$; the 95% bootstrap bias corrected confidence interval was $[.16, .90]$). A bootstrapping full analysis, based upon 100 resamplings with 10 observations, revealed that the following multiple, stepwise type ($p_{in} = .05$, $p_{out} = .055$), linear regression model

$$CPSP = a + b\,MSC_{avr} + c\,MSC_{dis} + d\,CE_{avr} + e\,CE_{dis},$$
$$\text{with constant } a,\ b,\ c,\ d \text{ and } e \text{ such that}$$
$$a \neq 0, \quad b, d \geqslant 0 \quad \text{and} \quad c, e \leqslant 0,$$

applied in 60% of these hundred resamplings (the 95% confidence interval for this count was $[50, 69]$[3]; that only MSC_{avr} was significant predictor of CPSP occurred in 35 of these 60 resamplings[4]). While wrong models (where $c > 0$, for example) were obtained in 23% of the resamplings, such a model was not applicable in 17% of them.

The correlations among the utilized variables for the pairs of the average subjects ($N = 11$) are presented in Table 3. One of these Pearson's correlation coefficients was significant at a .05 level. Due to insignificant correlations of MSC_{avr}, MSC_{dis}, CE_{avr} and CE_{dis} with CPSP, a multiple, stepwise type, linear regression analysis with CPSP as independent variable and MSC_{avr}, MSC_{dis}, CE_{avr} and CE_{dis} as dependent variables was not performed.

Table 3. Correlations among the utilized variables for the pairs of the average subjects

VARIABLE	2	3	4	5
1. CPSP	.32	−.15	.40	−.16
2. MSC_{avr}		−.35	.51	−.00
3. MSC_{dis}			−.40	.35
4. CE_{avr}				−.71*
5. CE_{dis}				

$^{*}p < .05$

For the average pairs' data, a bootstrapping, two-variable analysis, based upon 1000 resamplings with 11 observations, was undertaken. It revealed that none of variables MSC_{avr}, MSC_{dis}, CE_{avr} and CE_{dis} was correlated with CPSP. Despite that, a bootstrapping full analysis, based upon 100 resamplings with 11 observations, concerning the above-mentioned multiple linear regression model was performed. It evidenced that such a model applied in 35% of these hundred cases (the 95% confidence interval for this count was $[26, 45]$; that only MSC_{avr} was significant predictor occurred in 21 of these 35 resamplings[5]). While wrong models were obtained in 10% of these hundred cases, such a model was not applicable in 55% of them.

[3]obtained by utilizing calculator at `www.graphpad.com/quickcalcs/ConfInterval1.cfm`

[4]In 6 of 25 (60-35) remaining favorable cases model $CPSP = a + b\,MSC_{avr} + e\,CE_{dis}$ ($a \neq 0$, $b > 0$ and $e < 0$) applied, for example. Model $CPSP = a + d\,CE_{avr}$ ($a \neq 0$ and $d > 0$) was obtained in 9 resamplings.

[5]Model $CPSP = a + d\,CE_{avr}$ ($a \neq 0$ and $d > 0$) applied in 8 of 14 (35-21) remaining favorable cases, for example.

Discussion

Three important findings emerged from this study. First, collaborative problem solving performance was positively influenced by average mathematical self-concept for paired talented students. Second, the talented pairs' bootstrapped data evidenced that this performance could be explained by a multiple liner regression model, where average mathematical self-concept for paired students and average cognitive empathy for paired students had zero or positive influence, whereas absolute mathematical self-concept distance for paired students and absolute cognitive empathy distance for paired students had zero or negative effect. Third, though not supported, the validity of that model was indicated by the average pairs' bootstrapped data.

As mathematical self-concept seems to be an important predictor of mathematics achievement [13], we expected that collaborative problem solving performance would be positively influenced by average mathematical self-concept for paired students. Why was such a finding only obtained for the talented pairs? A two-independent-sample Kolmogorov-Smirnov test involving the utilized variables revealed that only for collaborative problem solving performance the two groups' data did not come from the same population (K-S $Z = 2.06$, $p < .001$). Since the talented pairs demonstrated a higher variance of collaborative problem solving performance (see Table 1), such a richer variable offered more chances for that relation, when applies, to be uncovered. To increase the variability of solvers' achievements enabling the discovery/confirmation of relations in question (especially for pairs comprising average and below average mathematics students), all potentially useful mathematical data (concepts, formulas, etc.) should be made available to solvers in the traditional or an electronic form.

As it was assumed that, contrary to higher averages of mathematical self-concept and cognitive empathy for paired students that would positively influence their problem solving performance, larger absolute distances in mathematical-self concept and cognitive empathy for paired students are likely to create specific affective and metacognitive tensions between students in pair reducing their collaborative problem solving performance (especially when problem is to be solved in several ways requiring higher and similar levels of mathematical self-concept and cognitive empathy), the above-mentioned regression model was validated. Because the size of both groups of collaborative solvers was quite small, we could only confidently validate it through bootstrapping regression analyses, the utilization of which was granted by the fact that both groups of pairs seemed to adequately represent their corresponding populations (talented vs. average). Since the outcomes of these analyses evidenced/indicated that this model may indeed hold true, further research may test and elaborate it. To achieve this end, the applied measure of cognitive empathy or perspective taking may be more related to collaborative problem solving. Furthermore, variables concerning cognitive style may be also considered bearing in mind that mixing students with different thinking styles can empower group learning (see [10]). Finally, other critical variables concerning the role of peer and instructional process influences on collaborative learning (see [16]) may be realized, operationalized and incorporated into an appropriate (regression or other) performance model.

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